Study of primary aberrations from the stigmatism theory

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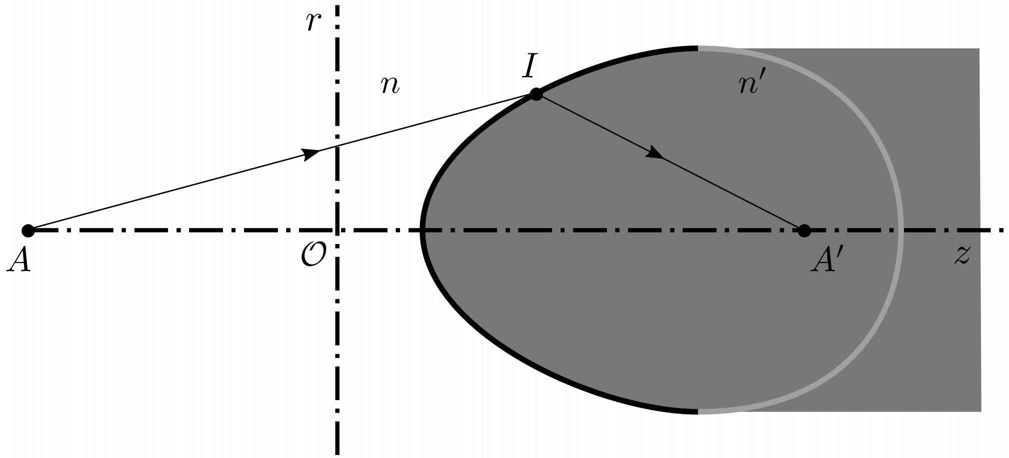
Cartesian surfaces

Are rigorously stigmatic surfaces able to produce a perfect point image from a point object on the optical axis.

Constant Optical Path Length (OPL)

$$\boxed{\mathrm{OPL} = \mathrm{n}\overline{AI} + n'\overline{IA'} = cte}$$



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[1] Descartes, R. Discours de la méthode pour bien conduire sa raison et chercher la vérité dans les sciences, with three appendices: La Dioptrique. (1637).
[2] Silva-Lora, A. & Torres, R. Explicit cartesian oval as a superconic surface for stigmatic imaging optical systems with real or virtual source or image. Proc. Royal Soc. A 476, 20190894 (2020).

Cartesian surfaces

Are rigorously stigmatic surfaces able from a point object on the optical axis

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Explicit Cartesian oval as a superconic surface for stigmatic imaging optical systems with real or virtual source or image

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Cartesian ovals, also known as rigorously stigmatic surfaces, are the simplest optical systems capable of producing a perfect point image. Exist both implicit and explicit expressions to represent these surfaces, but they treat both refractive and reflective surfaces independently. Because of the complexity of explicit expressions, the ray-tracing techniques for these surfaces are implemented using third-party software. In this paper, we express Cartesian ovals as a degenerated superconic curve and get a new explicit formulation for Cartesian ovals capable of treating image formation using both object and image points, either real or virtual, and in this formulation can deal with both reflective and refractive rigorously stigmatic surfaces. Finally, using the resultant expressions and the vector Snell–Descartes Law, we propose a self-contained analytical ray-tracing technique for all these surfaces.

[1] Descartes, R. Discours de la méthode pour bien conduire sa raison et chercher la vérité dans les sciences, with three appendices: La Dioptrique. (1637).
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Cartesian surfaces

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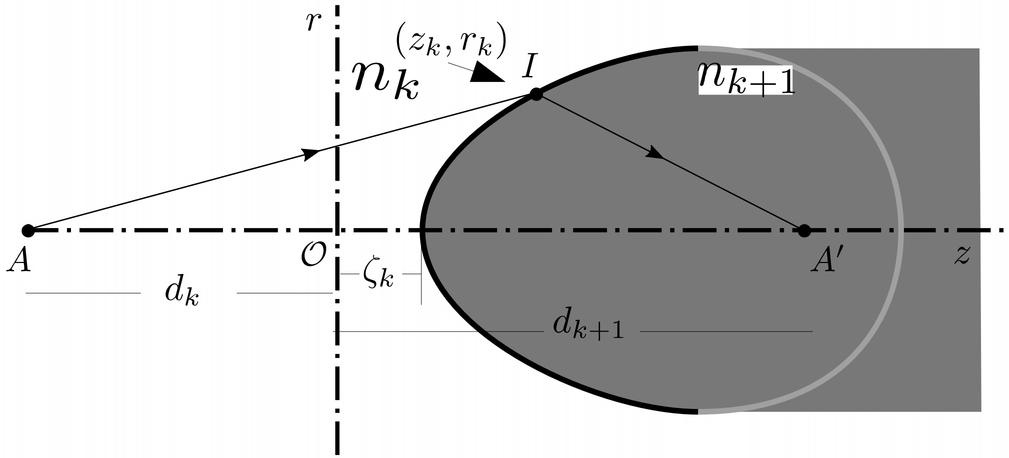
These surfaces can be represented by implicit mathematical equations

$$O_k G_k (z_k - \zeta_k)^2 - 2(1 + S_k \rho^2)(z_k - \zeta_k) + (O_k + T_k \rho_k^2)\rho_k^2 = 0$$

Or explicit ones

$$z_k(\rho_k) = \zeta_k + \frac{(O_k + T_k \rho_k^2)\rho_k^2}{1 + S_k \rho_k^2 + \sqrt{1 + (2S_k - O_k^2 G_k)\rho_k^2}}$$

where $\rho_k = \sqrt{(z_k - \zeta_k)^2 + r_k^2}$



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Cartesian surfaces

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$$O_kG_k(z_k-\zeta_k)^2-2(1+S_k\rho^2)(z_k-\zeta_k)+(O_k+T_k\rho_k^2)\rho_k^2=0$$

Or explicit ones

$$z_k(\rho_k)=\zeta_k+\frac{(O_k+T_k\rho_k^2)\rho_k^2}{1+S_k\rho_k^2+\sqrt{1+(2S_k-O_k^2G_k)\rho_k^2}}$$

where $\rho_k=\sqrt{(z_k-\zeta_k)^2+r_k^2}$

Form parameters

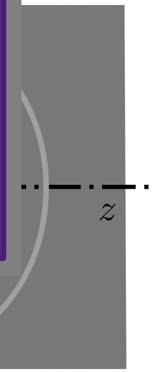
$$G_k=\frac{\left(\frac{n_{k+1}^2}{d_k-\zeta_k}-\frac{n_k^2}{d_{k+1}-\zeta_k}\right)^2}{n_kn_{k+1}\left(\frac{n_{k+1}}{d_{k+1}-\zeta_k}-\frac{n_k}{d_k-\zeta_k}\right)\left(\frac{n_{k+1}}{d_k-\zeta_k}-\frac{n_k}{d_{k+1}-\zeta_k}\right)}$$

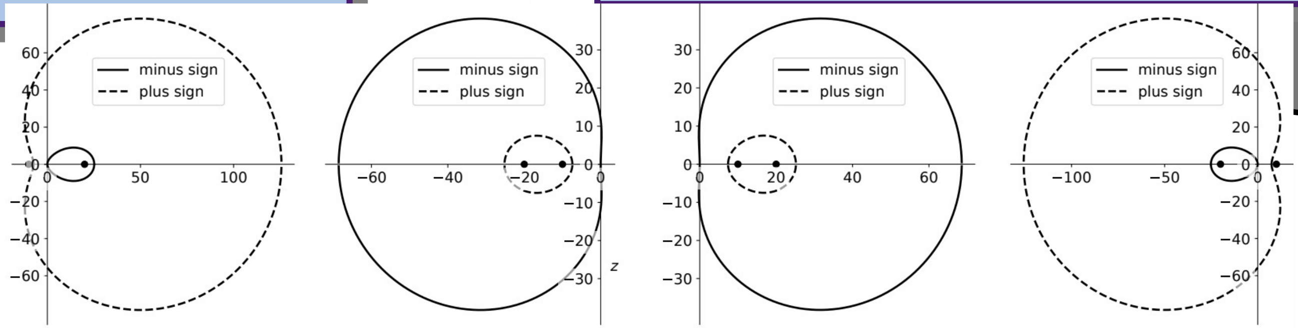
$$O_k=\frac{\frac{n_{k+1}}{(d_{k+1}-\zeta_k)}-\frac{n_k}{(d_k-\zeta_k)}}{(n_{k+1}-n_k)}$$

$$T_k=\frac{\left(\frac{n_{k+1}+n_k}{(d_{k+1}-\zeta_k)(d_k-\zeta_k)}\right)^2(n_{k+1}-n_k)}{4n_{k+1}n_k\left(\frac{n_{k+1}}{(d_k-\zeta_k)}-\frac{n_k}{(d_{k+1}-\zeta_k)}\right)}$$

$$S_k=\frac{\frac{(n_{k+1}+n_k)}{(d_{k+1}-\zeta_k)(d_k-\zeta_k)}\left(\frac{n_{k+1}^2}{(d_k-\zeta_k)}-\frac{n_k^2}{(d_{k+1}-\zeta_k)}\right)}{2n_{k+1}n_k\left(\frac{n_{k+1}}{(d_k-\zeta_k)}-\frac{n_k}{(d_{k+1}-\zeta_k)}\right)}$$

$$S_k^2=G_kO_kT_k$$





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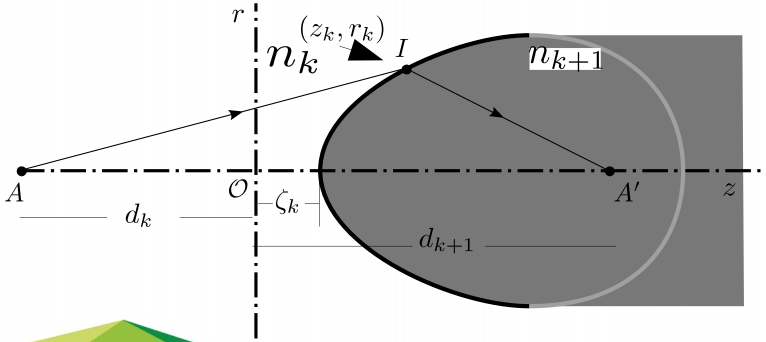
Cartesian surfaces

The conics surfaces are particular cases of Cartesian surfaces (ellipsoids, hyperboloids, paraboloids y spheres).

Schwarzschild equation

$$z_k(r_k) = \zeta_k + \frac{O_k r_k^2}{1 + \sqrt{1 - (G_k + 1) O_k^2 r_k^2}}$$



Condition

$T_k = S_k = 0$

valid to $d_k \to -\infty$ or $d_{k+1} \to \infty$ — **Refractive surfaces**

valid to $n_{k+1} = -n_k$ — **Reflective surfaces**

[1] Silva-Lora, A. & Torres, R. Superconical aplanatic ovoid singlet lenses. JOSA A 37, 1155–1165 (2020).

Cartesian surfaces

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The conics surfaces are particular cases of Cartesian surfaces (ellipsoids, hyperboloids, paraboloids y **spheres**).

Condition

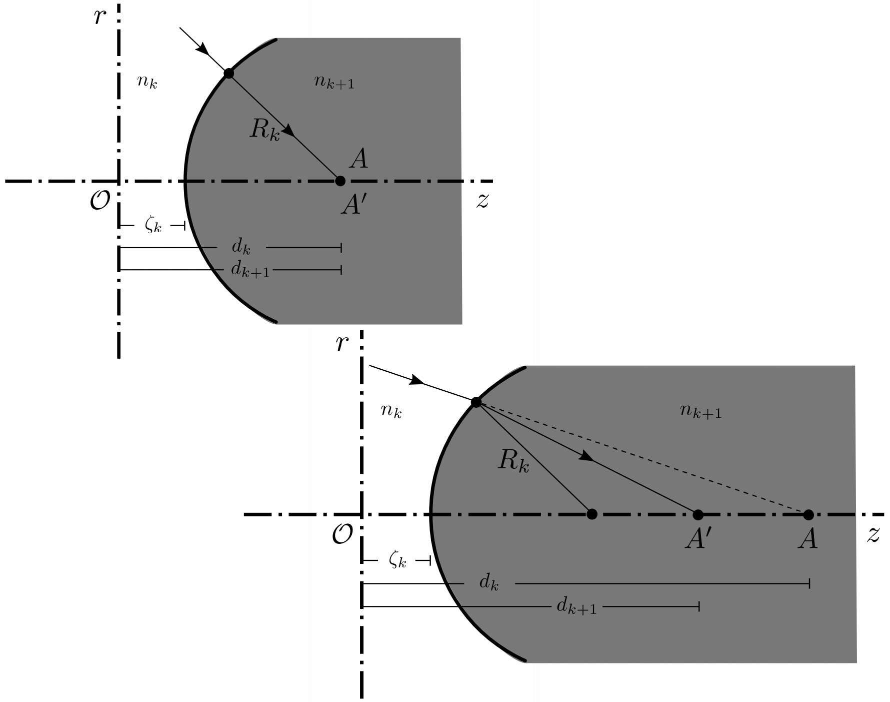
Sphere with $d_{k+1} = d_k$

Sphere with radius R_k

$$d_k = \zeta_k + \frac{R_k(n_{k+1} - n_k)}{n_k}$$

$$d_{k+1} = \zeta_k + \frac{R_k(n_{k+1} - n_k)}{n_{k+1}}$$

Spherical Refracting surfaces

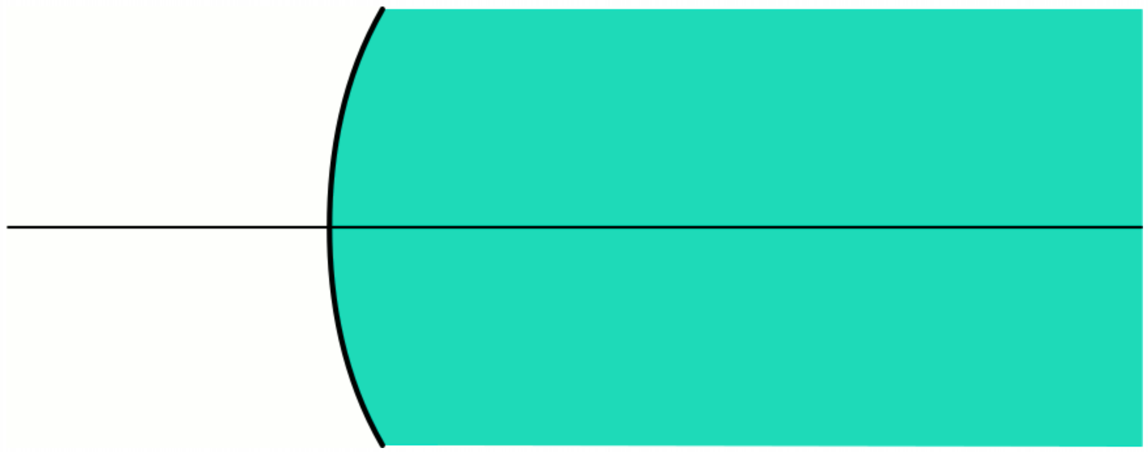


[1] Silva-Lora, A. & Torres, R. Superconical aplanatic ovoid singlet lenses. JOSA A 37, 1155–1165 (2020).
[2] Young T. 1807 Lectures on natural philosophy. London 1, 464.

Cartesian surfaces

Are cataloged as aspherical surfaces but they are not expressed as a standard formulation

Spherical surface



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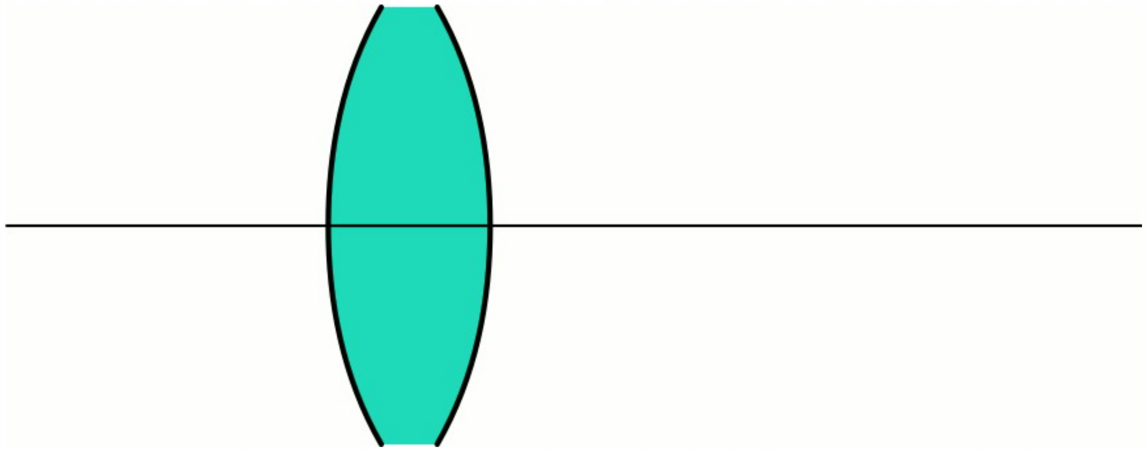
Spherical lens



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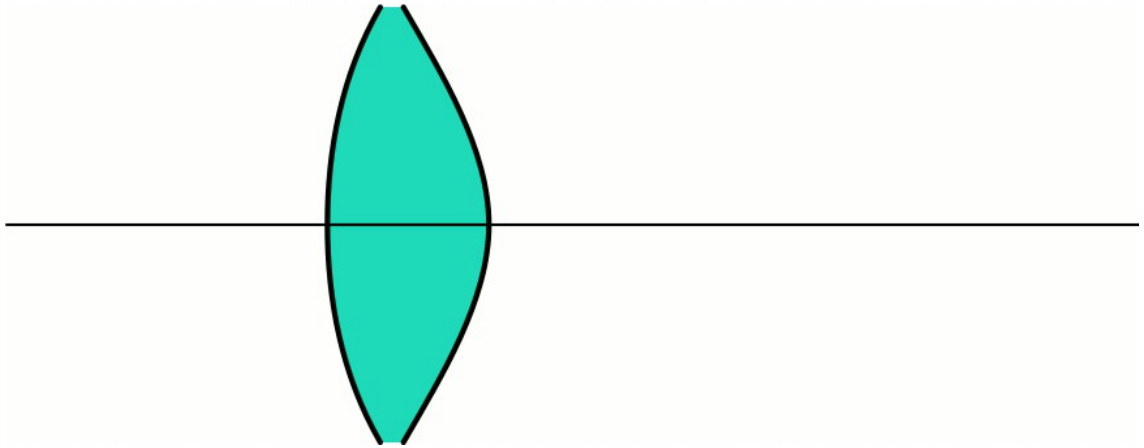
Aspherical lens



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[2] Silva-Lora, A., & Torres, R. (2020). Superconical aplanatic ovoid singlet lenses. JOSA A, 37(7), 1155-1165.

Cartesian surfaces

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Are cataloged as aspherical surfaces but they are not expressed as a standard formulation

Standard aspherical

$$z_k(r_k) = \underbrace{\zeta_k + \frac{O_k r_k^2}{1+\sqrt{1-(G_k+1)O_k^2 r_k^2}}}_{\text{Conic base}} + \sum_{n=2}^{\infty} \underbrace{A_{2n}}_{\text{Aspheric coefficients}} r_k^{2n}$$

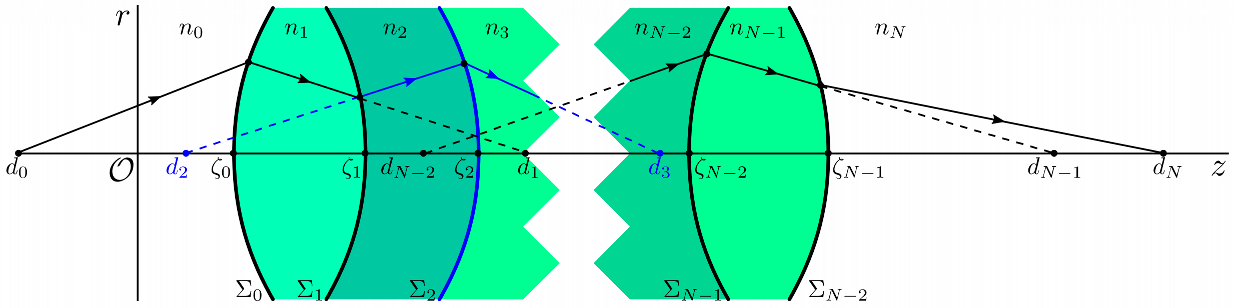
Conic base **Aspheric coefficients**

Cartesian surfaces

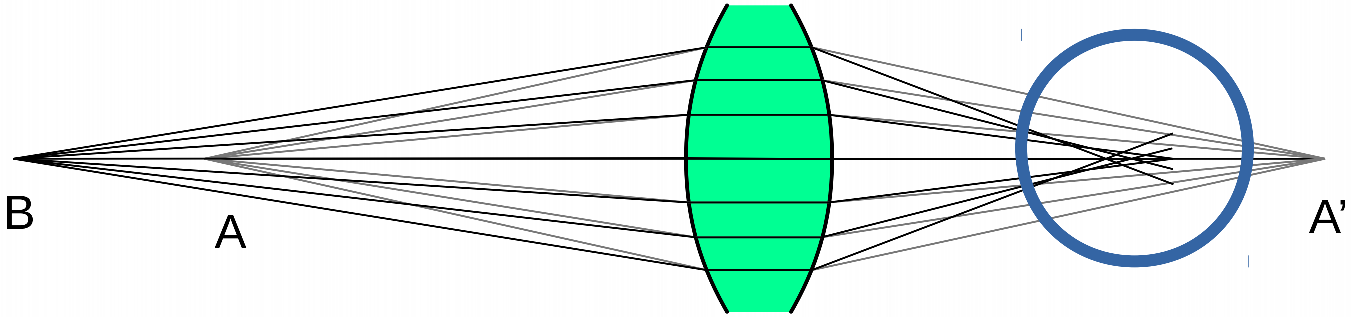
$$z_k(\rho_k) = \zeta_k + \frac{(O_k + T_k\rho_k^2)\rho_k^2}{1 + S_k\rho_k^2 + \sqrt{1+(2S_k - O_k^2 G_k)\rho_k^2}}$$

Ray-tracing through Cartesian surfaces

- A series of systems can be designed using a set of Cartesian surfaces
- Rigorous stigmatism is preserved through each surface
- Aplanatism not warranted



Optical system



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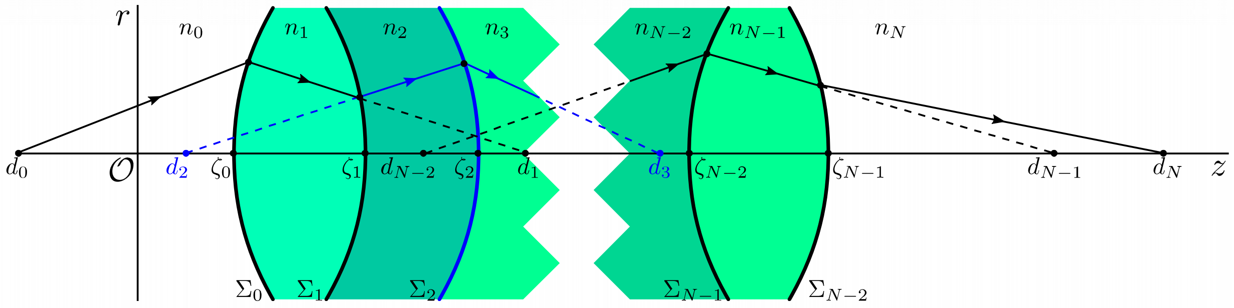
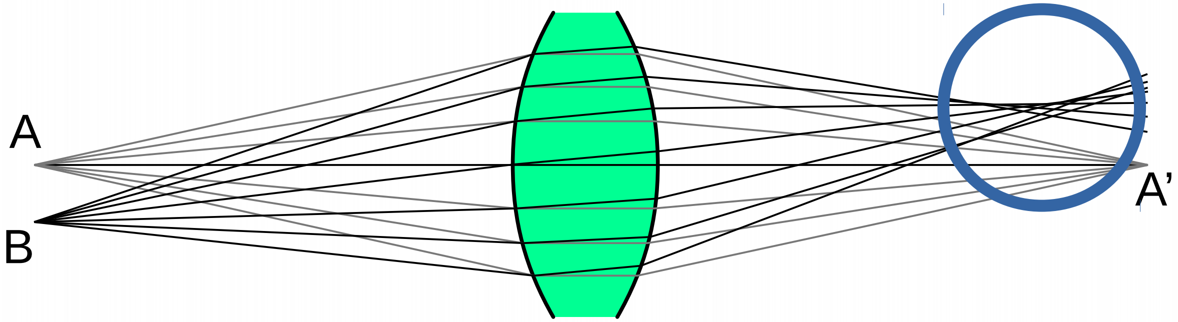
Ray-tracing through Cartesian surfaces

- A series of systems can be designed using a set of Cartesian surfaces
- Rigorous stigmatism is preserved through each surface
- Aberrations due to displacements of stigmatic points



Optical system



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Ray-tracing through Cartesian surfaces

Ray-tracing technique can be achieved by developing two main steps:

- Find the intersection between a ray and a Cartesian surface
- Apply Snell-Descartes law

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Ray-tracing through Cartesian surfaces

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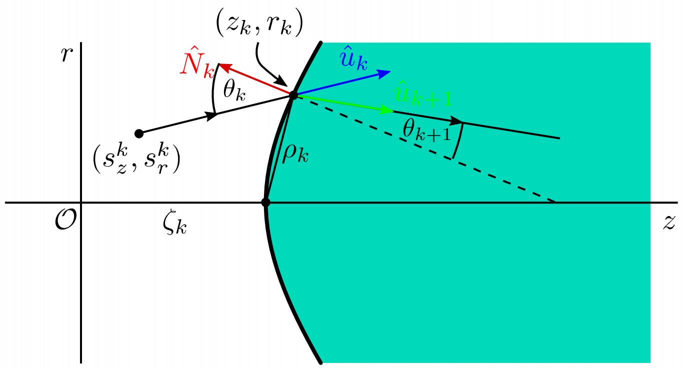
Intersection between a ray and a Cartesian surface



Equation of a line

$x_k = \frac{u_x^k}{u_z^k}(z_k - \zeta_k) + b_x^k$ where $b_x^k = \frac{u_x^k}{u_z^k}(z_k - \zeta_k) + s_x^k$

$y_k = \frac{u_y^k}{u_z^k}(z_k - \zeta_k) + b_y^k$ where $b_y^k = \frac{u_y^k}{u_z^k}(z_k - \zeta_k) + s_y^k$

$z_k = z_k$

Source

(s_x^k, s_y^k, s_z^k)

Incident unit vector

$\hat{u}_k = (u_x^k, u_y^k, u_z^k)$

$$\rho_k^2 = (z_k - \zeta_k)^2 + r_k^2 = (z_k - \zeta_k)^2 + \left(b_x^k + (z_k - \zeta_k)\frac{u_x^k}{u_z^k}\right)^2 + \left(b_y^k + (z_k - \zeta_k)\frac{u_y^k}{u_z^k}\right)^2$$

Replacing

Implicit expression for Cartesian surfaces

$$G_k O_k (z_k - \zeta_k)^2 - 2(z_k - \zeta_k)\left(1 + S_k \rho_k^2\right) + \rho_k^2\left(O_k + T_k \rho_k^2\right) = 0$$

[1] Silva-Lora, A. & Torres, R. Superconical aplanatic ovoid singlet lenses. JOSA A 37, 1155–1165 (2020).

Ray-tracing through Cartesian surfaces

It is obtained a quartic expression which can be solved using Ferrari method.

$$Q_4(z_k-\zeta_k)^4+Q_3(z_k-\zeta_k)^3+Q_2(z_k-\zeta_k)^2+Q_1(z_k-\zeta_k)+Q_0=0$$

$$Q_0=\left(O_k+T_k\left(b_x^2+b_y^2\right)\right)\left(b_x^2+b_y^2\right)$$

$$Q_1=2O_k\left(b_xu_{xz}+b_yu_{yz}\right)-2S_k\left(b_x^2+b_y^2\right)+4T_k\left(b_x^2+b_y^2\right)\left(b_xu_{xz}+b_yu_{yz}\right)-2$$

$$Q_2=G_kO_k+\frac{O_k}{u_z^2}-4S_k\left(b_xu_{xz}+b_yu_{yz}\right)+4T_k\left(b_xu_{xz}+b_yu_{yz}\right)^2+\frac{2T_k\left(b_x^2+b_y^2\right)}{u_z^2}$$

$$Q_3=\frac{-2S_k+4T_k\left(b_xu_{xz}+b_yu_{yz}\right)}{u_z^2}$$

$$Q_4=\frac{T_k}{(u_z^k)^4}$$

From here is obtained the axial coordinate of the intersection z_k

x_k and y_k can be obtained using the equation of the line

[1] Silva-Lora, A. & Torres, R. Superconical aplanatic ovoid singlet lenses. JOSA A 37, 1155–1165 (2020).

Ray-tracing through Cartesian surfaces

To carry out the ray-tracing method it is necessary some elements. From

$$f_k(x_k, y_k, z_k) = G_k O_k (z_k - \zeta_k)^2 - 2(z_k - \zeta_k)\left(1 + S_k \rho_k^2\right) + \rho_k^2 \left(O_k + T_k \rho_k^2\right)$$

Are obtained

$$\frac{\partial f_k}{\partial x_k} = 2x_k(O_k - 2S_k(z_k - \zeta_k) + 2T_k\rho_k^2)$$

$$\frac{\partial f_k}{\partial z_k} = 2(z_k - \zeta_k)(O_k - 2S_k(z_k - \zeta_k) + 2T_k\rho_k^2 + G_k O_k) - 2S_k\rho_k^2 - 2$$

$$\frac{\partial f_k}{\partial y_k} = 2y_k(O_k - 2S_k(z_k - \zeta_k) + 2T_k\rho_k^2)$$

Normal unit vector

$$\hat{N}_k = \frac{\nabla f_k(x_k, y_k, z_k)}{|\nabla f_k(x_k, y_k, z_k)|} = \frac{\frac{\partial f_k}{\partial x_k}\hat{i} + \frac{\partial f_k}{\partial y_k}\hat{j} + \frac{\partial f_k}{\partial z_k}\hat{k}}{\left(\frac{\partial f_k}{\partial x_k}\right)^2 + \left(\frac{\partial f_k}{\partial y_k}\right)^2 + \left(\frac{\partial f_k}{\partial z_k}\right)^2}$$

[1] Silva-Lora, A. & Torres, R. Superconical aplanatic ovoid singlet lenses. JOSA A 37, 1155–1165 (2020).

Ray-tracing through Cartesian surfaces

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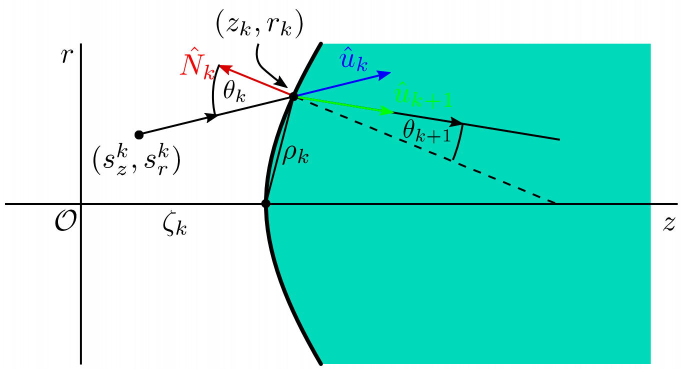
These elements are used to find the refracted unit vector using the vector form of the Snell-Descartes law.

Snell-Descartes Law

$$n_k \sin\theta_k = n_{k+1} \sin\theta_{k+1}$$



Vector form of Snell-Descartes Law

$$\hat{u}_{k+1} = \frac{n_k}{n_{k+1}}\hat{u}_k - \left[\frac{n_k}{n_{k+1}}(\hat{N}_k \cdot \hat{u}_k) + \sqrt{1 - \frac{n_k^2}{n_{k+1}^2}\left(1 - (\hat{N}_k \cdot \hat{u}_k)^2\right)}\right]$$

For a system composed of a set of Cartesian surfaces this process is repeated until ray reaches the image plane.

[1] Silva-Lora, A. & Torres, R. Superconical aplanatic ovoid singlet lenses. JOSA A 37, 1155–1165 (2020).

Rigorously Stigmatic Lenses (RSL)

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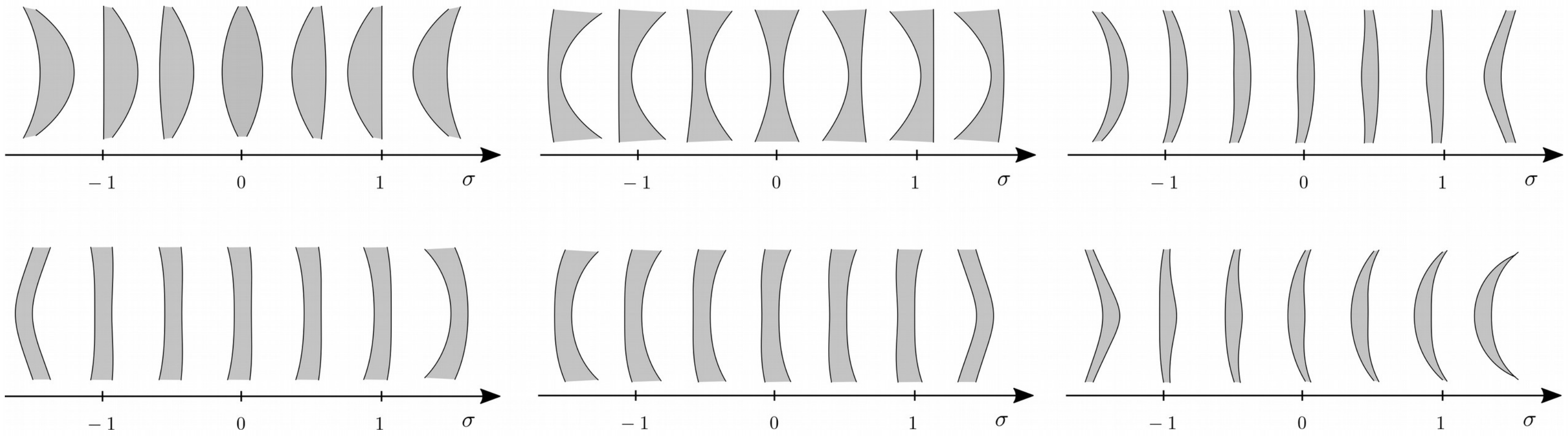
Are produced using two or more Cartesian surfaces or using aspherical corrective surfaces.

Lenses composed of two surfaces (singletes)



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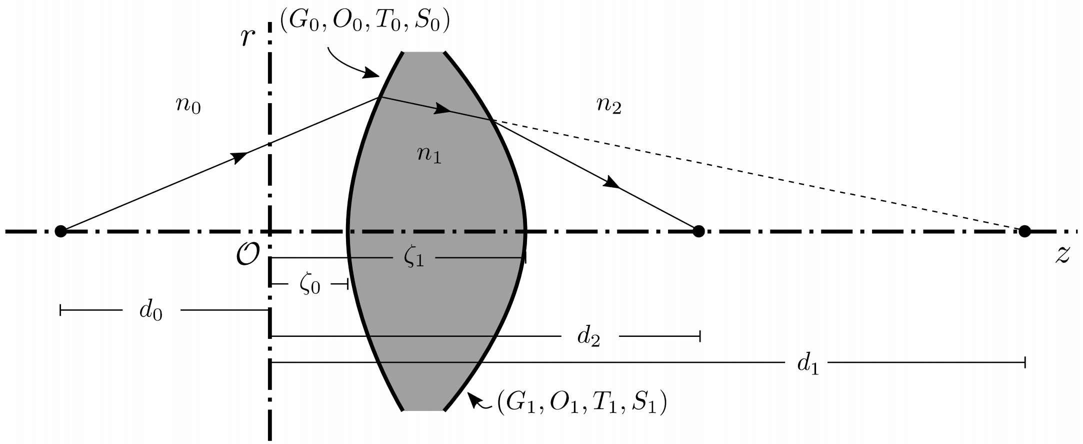
Rigorously Stigmatic Lenses (RSL)

There are infinite shapes for singlets composed of Cartesian surfaces, characterized by a shape factor $\boldsymbol{\sigma}$



Paraxial curvature

$$O_0 = \frac{\frac{n_1}{(d_1-\zeta_0)} - \frac{n_0}{(d_0-\zeta_0)}}{(n_1 - n_0)}$$

$$O_1 = \frac{\frac{n_2}{(d_2-\zeta_1)} - \frac{n_1}{(d_1-\zeta_1)}}{(n_2 - n_1)}$$

Shape factor

$$\sigma = \frac{O_0 + O_1}{O_0 - O_1} = -\frac{\left(\frac{n_1}{d_1-\zeta_1} - \frac{n_2}{d_2-\zeta_1}\right) + \frac{(n_1-n_2)}{(n_1-n_0)}\left(\frac{n_1}{d_1-\zeta_0} - \frac{n_0}{d_0-\zeta_0}\right)}{\left(\frac{n_1}{d_1-\zeta_1} - \frac{n_2}{d_2-\zeta_1}\right) - \frac{(n_1-n_2)}{(n_1-n_0)}\left(\frac{n_1}{d_1-\zeta_0} - \frac{n_0}{d_0-\zeta_0}\right)}$$

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Rigorously Stigmatic Lenses (RSL)

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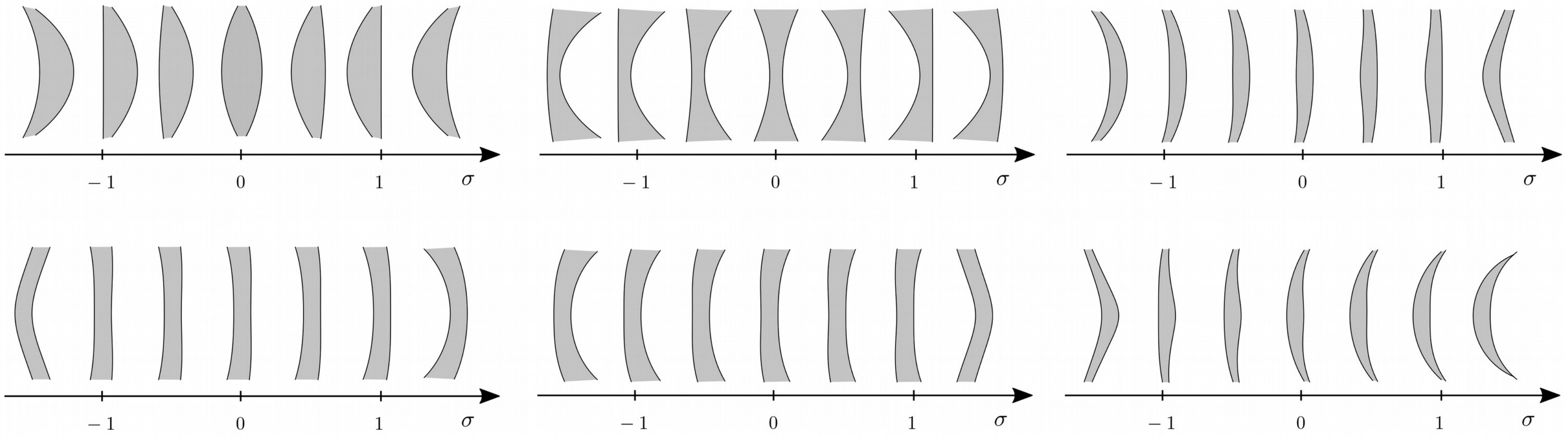
Shape factor

$$\sigma = \frac{O_0 + O_1}{O_0 - O_1} = -\frac{\left(\frac{n_1}{d_1-\zeta_1} - \frac{n_2}{d_2-\zeta_1}\right) + \frac{(n_1-n_2)}{(n_1-n_0)}\left(\frac{n_1}{d_1-\zeta_0} - \frac{n_0}{d_0-\zeta_0}\right)}{\left(\frac{n_1}{d_1-\zeta_1} - \frac{n_2}{d_2-\zeta_1}\right) - \frac{(n_1-n_2)}{(n_1-n_0)}\left(\frac{n_1}{d_1-\zeta_0} - \frac{n_0}{d_0-\zeta_0}\right)}$$



[1] Silva-Lora, A. & Torres, R. Superconical aplanatic ovoid singlet lenses. JOSA A 37, 1155–1165 (2020).
[2] T. T. Smith, "Spherical aberration in thin lenses," Phys. Rev. 19, 276 (1922).

Rigorously Stigmatic Lenses (RSL)

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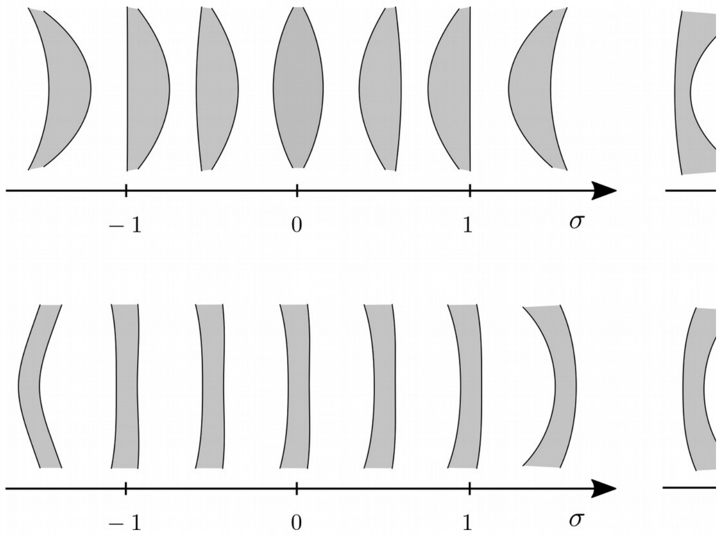
Shape factor



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OPTICS, IMAGE SCIENCE, AND VISION

Superconical aplanatic ovoid singlet lenses

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In this work, we return to Descartes's idea to develop a formalism to construct rigorously stigmatic singlet lenses comprising two Cartesian surfaces. Optical systems are built using a considerable number of spherical surfaces, presenting in most cases spherical aberration. Wasermann and Wolf proposed eliminating spherical aberration and minimizing third-order coma by using two adjacent aspherical surfaces. That is why, using a parametric formulation for Cartesian ovals, we propose the design of singlet lenses where the condition of rigorous stigmatism is guaranteed for each surface, and therefore, strictly speaking, in the pair of stigmatic points, the lens becomes an optical system free of spherical aberration. This formulation is unified to both refractive and reflective optical surfaces. Therefore, within the framework of the theory of rigorously stigmatic optical systems, making use of Cartesian surfaces for the construction of stigmatic ovoid singlet lenses, we achieve the same functionality of optical systems involving a set of spherical lenses. These lenses have the advantage of being formulated according to a generalized shape factor associated with the Coddington shape factor, allowing an easy classification of these stigmatic lenses. The ideal imaging is carried out by applying an exact ray-tracing method through these ovoid singlet lenses.

https://doi.org/10.1364/JOSAA.392795

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[1] Silva-Lora, A. & Torres, R. Superconical aplanatic ovoid singlet lenses. JOSA A 37, 1155–1165 (2020).
[2] T. T. Smith, "Spherical aberration in thin lenses," Phys. Rev. 19, 276 (1922).

Aplanatism in RSL

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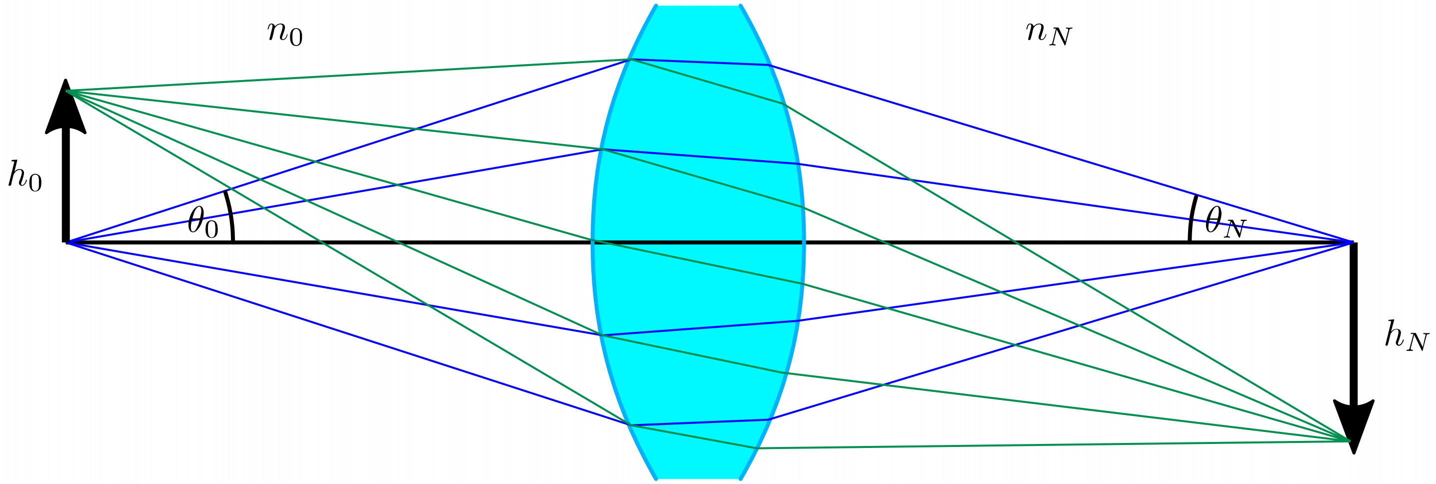
Aplanatism refers to the property of lenses to form images of extended objects.



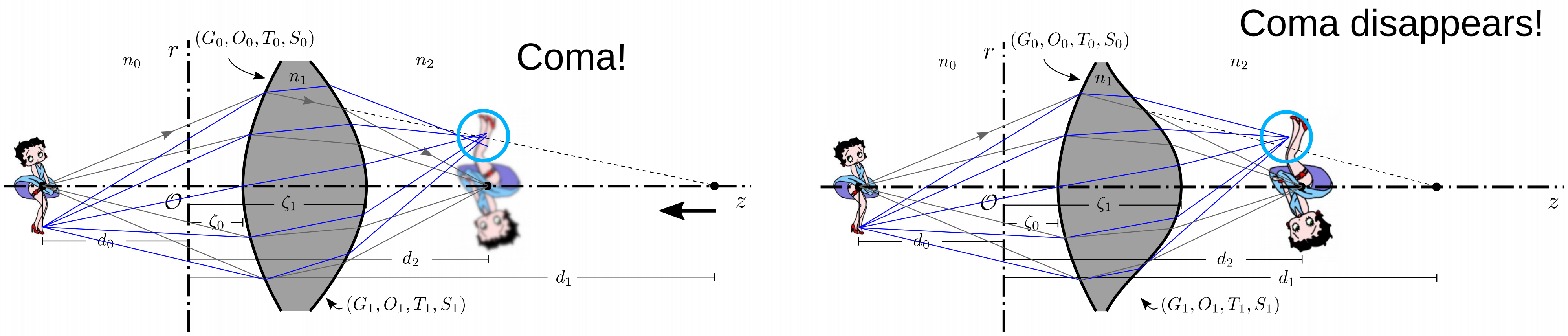
Abbe sine condition

$$\frac{\sin\theta_0}{\sin\theta_N} = -\frac{n_N}{n_0}\frac{h_N}{h_0}$$

[1] Silva-Lora, A., & Torres, R. (2020). Aplanatism in stigmatic optical systems. Optics Letters, 45(23), 6390-6393.

Aplanatism in RSL

There are infinite Cartesian surfaces that combined each other can form an stigmatic singlet lens, but there is a specific pair of these surfaces that combined minimize the coma.



[1] Silva-Lora, A., & Torres, R. (2020). Aplanatism in stigmatic optical systems. Optics Letters, 45(23), 6390-6393.

Aplanatism in RSL

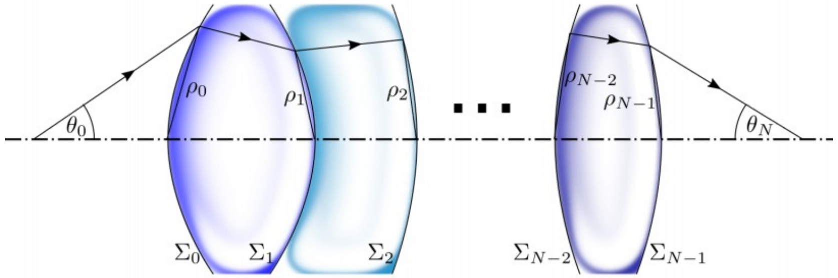
Systems composed of Cartesian surfaces can have the property of aplanatism under certain conditions.

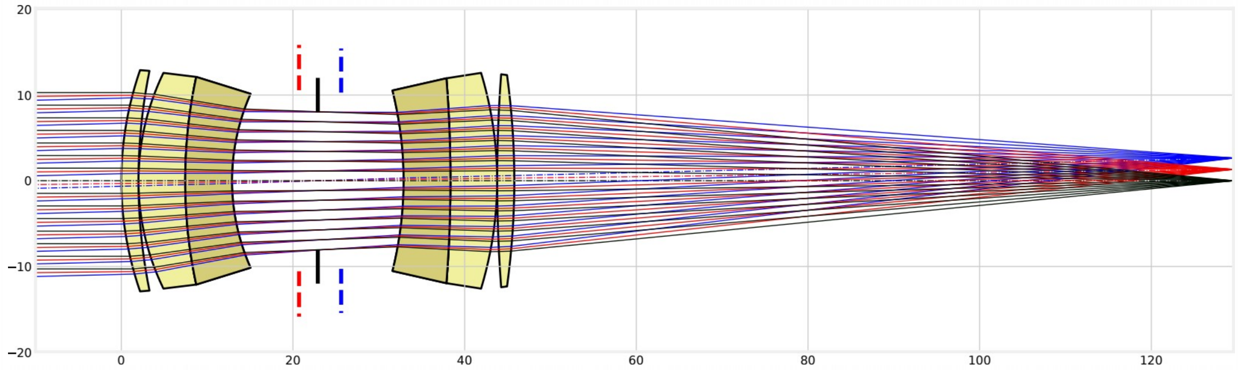
Abbe sine condition

$$\frac{\sin\theta_0}{\sin\theta_N} = \frac{n_N}{n_0} M\mathcal{M}(\rho_k), \quad \text{with} \quad \mathcal{M}(\rho_k) = 1$$

Deviation from Abbe sine condition

$$\mathcal{M}(\{\rho_k\}) = \prod_{k=0}^{N-1} \frac{n_{k+1}}{n_k} \frac{2S_k - G_kO_k^2 - \left(2S_k - \frac{G_kO_k}{d_{k+1}-\zeta_k}\right)\sqrt{1+(2S_k - G_kO_k^2)\,\rho_k^2}}{2S_k - G_kO_k^2 - \left(2S_k - \frac{G_kO_k}{d_k-\zeta_k}\right)\sqrt{1+(2S_k - G_kO_k^2)\,\rho_k^2}}$$





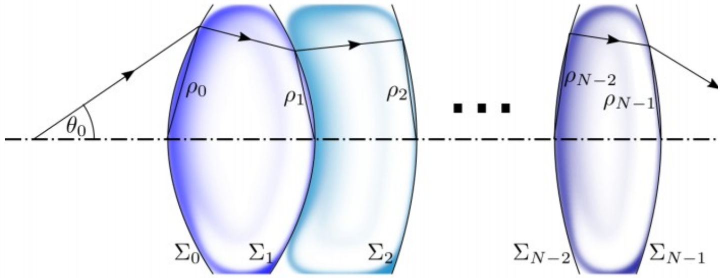
[1] Silva-Lora, A., & Torres, R. (2020). Aplanatism in stigmatic optical systems. Optics Letters, 45(23), 6390-6393.

Aplanatism in RS

Systems composed of C
aplanatism under certain c

Abbe sine condition

$$\frac{\sin\theta_0}{\sin\theta_N} = \frac{n_N}{n_0} M\mathcal{M}(\rho_k), \quad \text{with} \quad \mathcal{M}(\rho_k) =$$



Optics Letters

Aplanatism in stigmatic optical systems

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The minimization of spherical and coma aberrations in optical imaging systems is currently accomplished through the use of corrective aspheric optical surfaces. In this work, we develop a new, to the best of our knowledge, theory for the design of rigorously aplanatic optical systems, considering as a starting point the rigorous stigmatism theory of optical systems composed of Cartesian surfaces. The main characteristic of these surfaces is their, *a priori*, zero spherical aberration. In a general parametric formulation for systems made up of a set of these surfaces, the Abbe sine condition is adapted to simultaneously obtain the stigmatism and aplanatism conditions. Thus, we achieved the design of optical systems that in theory are immune to both coma and spherical aberrations.

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to fulfilling the Abbe sine condition, to the development of aplanatic optical systems in a general framework. To achieve this, we use a parametric expression for Cartesian surfaces and a set of shapes parameters that characterize them, both elements developed in previous works [18,19].

The theory of stigmatic optical systems is based on Descartes' ovoids, illustrated in Fig. 1(a). These ovoids, Σ_0, are surfaces of revolution, with vertex at V_0, that separate two media of refractive indices n_0 and n_1, which from the point object at P_0 produce a perfect image point at P_1. The proper combination of two of these surfaces allows us to design *stigmatic ovoid singlet lenses* (SOSL) [19] as is illustrated in Fig. 1(b). A combination of N surfaces produce in general stigmatic ovoid lenses (SOL), as shown in Fig. 1(c), guaranteeing the conservation of the stigmatism condition as light passes through each surface. The main feature of these systems is that all rays that pass through the

−20 0 20 40 60 80 100 120

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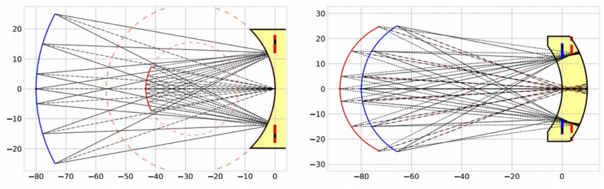
From equation of deviation from aplanatism condition can be obtained cases of strict aplanatism

$$\mathcal{M}(\{\rho_k\}) = \prod_{k=0}^{N-1} \frac{n_{k+1}}{n_k} \frac{2S_k - G_k O_k^2 - \left(2S_k - \frac{G_k O_k}{d_{k+1}-\zeta_k}\right)\sqrt{1+(2S_k - G_k O_k^2)\,\rho_k^2}}{2S_k - G_k O_k^2 - \left(2S_k - \frac{G_k O_k}{d_k-\zeta_k}\right)\sqrt{1+(2S_k - G_k O_k^2)\,\rho_k^2}} = 1$$

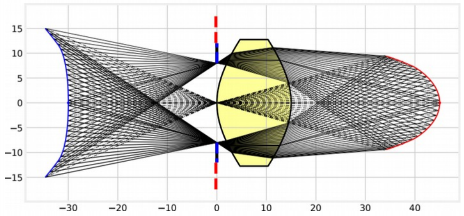
Type-0

$$2S_k - G_k O_k^2 = 0$$



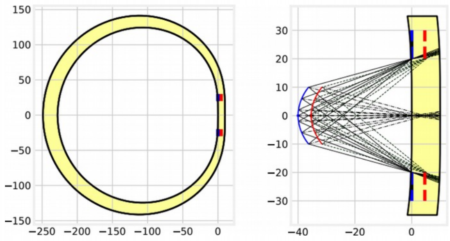
Type-1

$$N = 2, \quad d_1 \to \infty, \quad n_2 = n_0$$
$$d_2 - \zeta_1 = \pm(d_0 - \zeta_0)$$



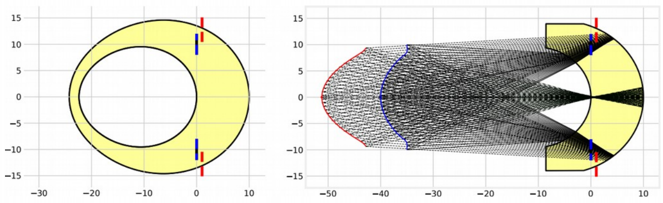
Type-2

$$O_k = 0$$



Type-3

$$G_k = 0$$



[1] Silva-Lora, A., & Torres, R. (2020). Aplanatism in stigmatic optical systems. Optics Letters, 45(23), 6390-6393.

Aplanatism in RSL

All RSL accomplish with this condition for $\{\rho_k\} \to 0$

$$\mathcal{M} = \prod_{k=0}^{N-1} \frac{n_{k+1}}{n_k} \frac{2S_k - G_k O_k^2 - \left(2S_k - \frac{G_k O_k}{d_{k+1} - \zeta_k}\right)}{2S_k - G_k O_k^2 - \left(2S_k - \frac{G_k O_k}{d_k - \zeta_k}\right)}$$

Paraxial invariant for RSL

$$\mathcal{M} = \prod_{k=0}^{N-1} \frac{n_{k+1}}{n_k} \left(\frac{O_k - \frac{1}{d_{k+1} - \zeta_k}}{O_k - \frac{1}{d_k - \zeta_k}} \right) = 1$$

Key to achromatism approach!

Applications

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There are some areas where this approach can be implemented

Microscopy

Astronomy

Photolithography

Ophthalmic

Illumination applications

Devices composed with Cartesian surfaces can be lighter due to they can have fewer surfaces than conventional ones.

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¡Gracias!