EL UNIVERSO AL ALCANCE DEL CÁLCULO

PARTE II
PROPAGACIÓN DE LA LUZ
LAS ECUACIONES DE LA COSMOLOGÍA

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CONSTANTES Y PARÁMETROS

Velocidad de la luz	$c = 3 \times 10^{10} cm/seg$
Constante gravitacional	$G = 6,67 \times 10^{-8} cm^3/grseg^2$
Parámetro de Hubble	$H_0 = 73 Km/segMpc$
Tiempo de Hubble	$H_0^{-1} = 13,8 \times 10^9 años$
Radio o longitud de Hubble	$L_{H_0} = 1,3 \times 10^{28} cm$
Densidad Total o crítica	$\rho_{T,0} \equiv \frac{3H_0^2}{8\pi G} \cong 9 \times 10^{-30} \frac{gr}{cm^3} \approx 5 protones/mt^3$
Omega de la materia	$\Omega_m \equiv \rho_{m,0}/\rho_T = 0.27$
Omega de la radiación	$\Omega_r \equiv \rho_{r,0}/\rho_T = 5 \times 10^{-5}$
Omega del vacío	$\Omega_v \equiv \rho_v/\rho_T = 0.73$

PROPAGACIÓN DE LUZ EN UN UNIVERSO EN EXPANSIÓN

Corrimiento al rojo

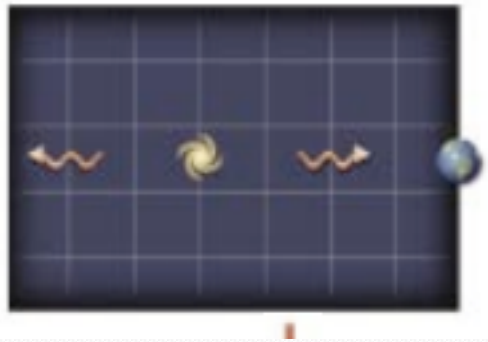
$$L \sim a(t) \Rightarrow \lambda \sim a(t) \Rightarrow \frac{\lambda_0}{\lambda} = a^{-1}(t)$$

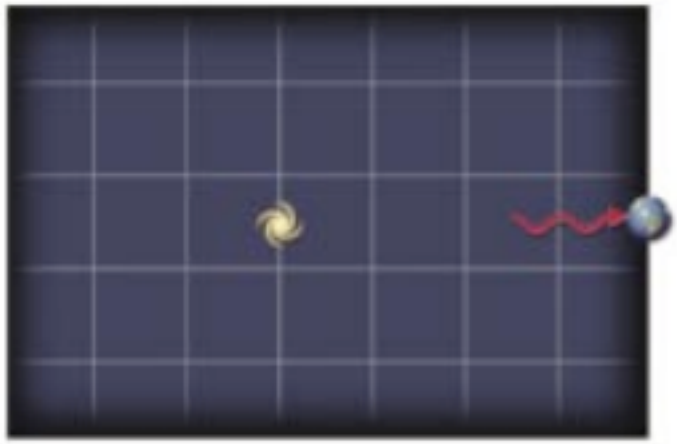
$$z = \frac{\lambda_0 - \lambda}{\lambda}$$

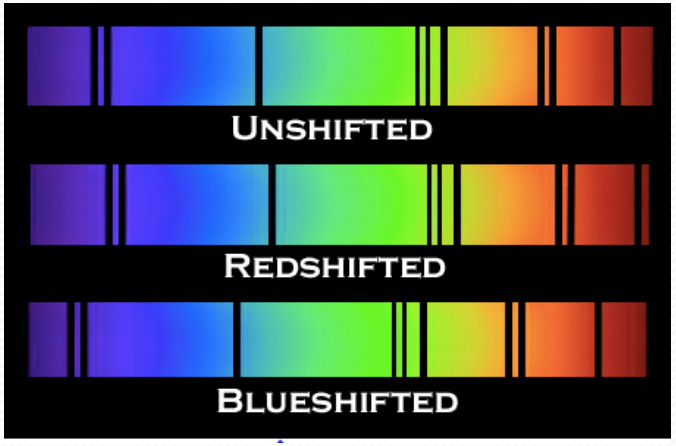
$$z + 1 = a^{-1}$$

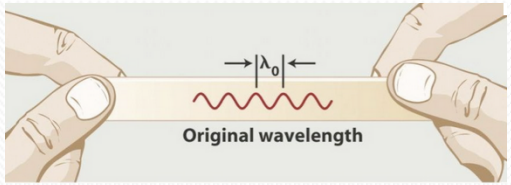
$$L = L_0 (z+1)^{-1}$$

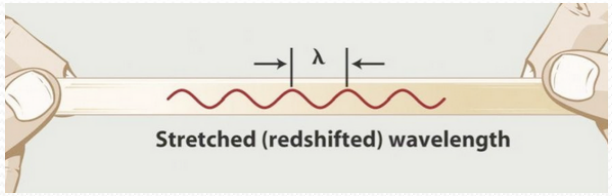
$$V = V_0 (z+1)^{-3}$$







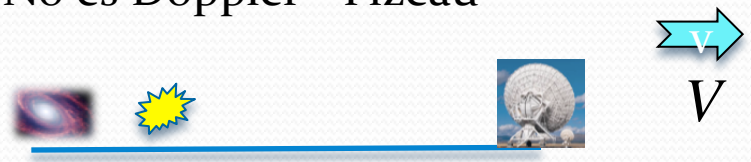




PROPAGACIÓN DE LUZ EN UN UNIVERSO EN EXPANSIÓN

$$z + 1 = a^{-1}$$

No es Doppler - Fizeau



Sí es expansión del espacio

$V = 0$

CÓMO MEDIR EL PARÁMETRO DE HUBBLE

Si Δt es pequeño: $a(t) \cong a(t_0) + \dot{a}(t_0)(t - t_0)$

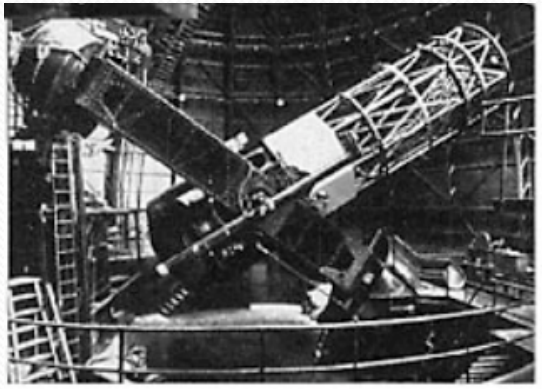
$$\frac{a(t)}{a(t_0)} = 1 + \frac{\dot{a}(t_0)}{a(t_0)}(t - t_0)$$

$$(z+1)^{-1} = 1 + H_0(t - t_0)$$

$$(1+z)^{-1} \approx 1 - z \qquad t_0 - t = \frac{L}{c} \qquad z = H_0 \frac{L}{c}$$

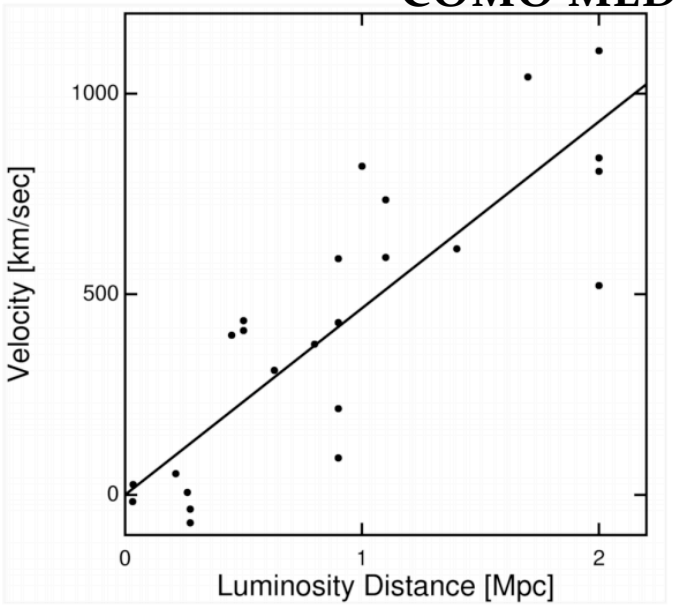
$$z = \frac{V}{c}$$

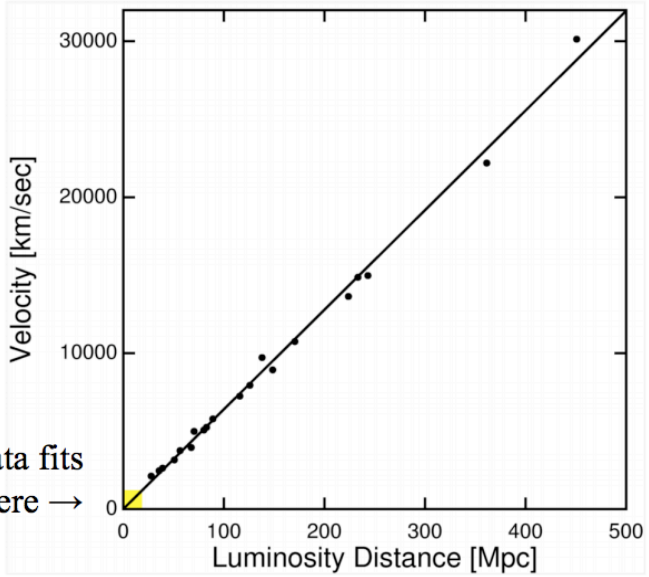
$$V = H_0 L$$

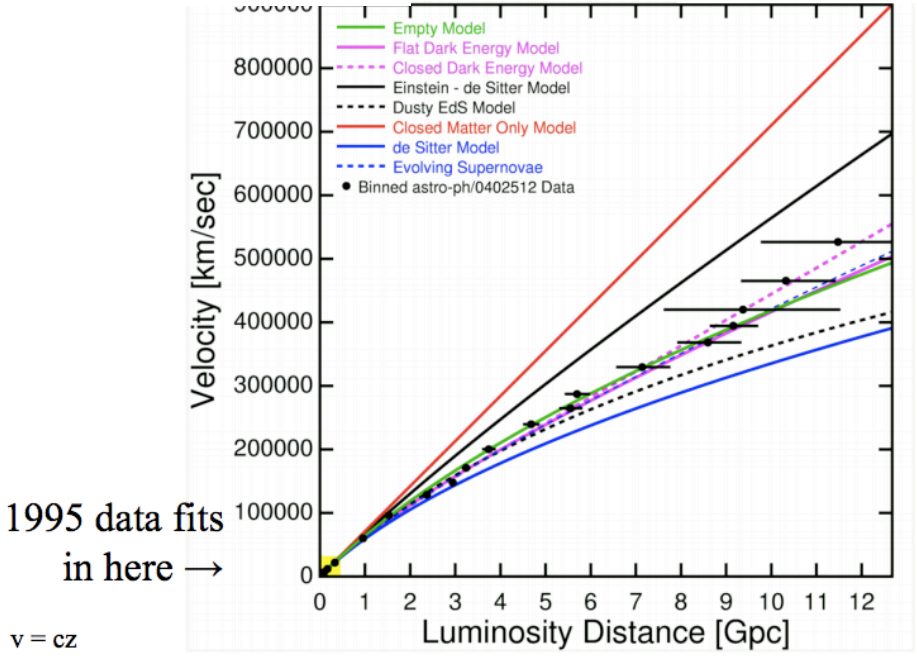


Mt. Wilson
100 Inch
Telescope

CÓMO MEDIR EL PARÁMETRO DE HUBBLE

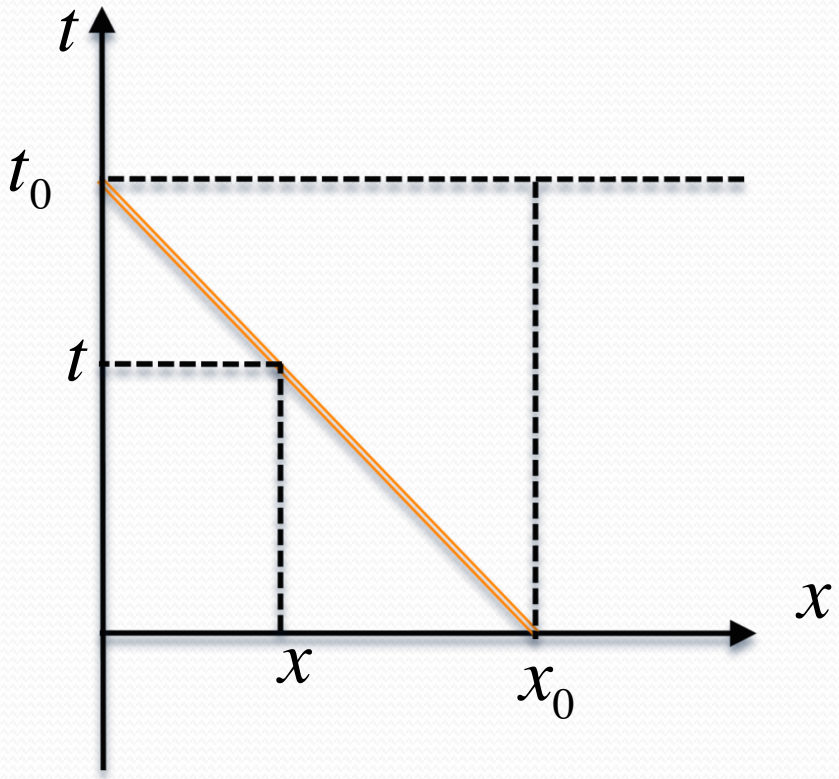






$$H_0 = 73 Km/segMpc$$

PROPAGACIÓN DE LUZ EN UN UNIVERSO ESTÁTICO



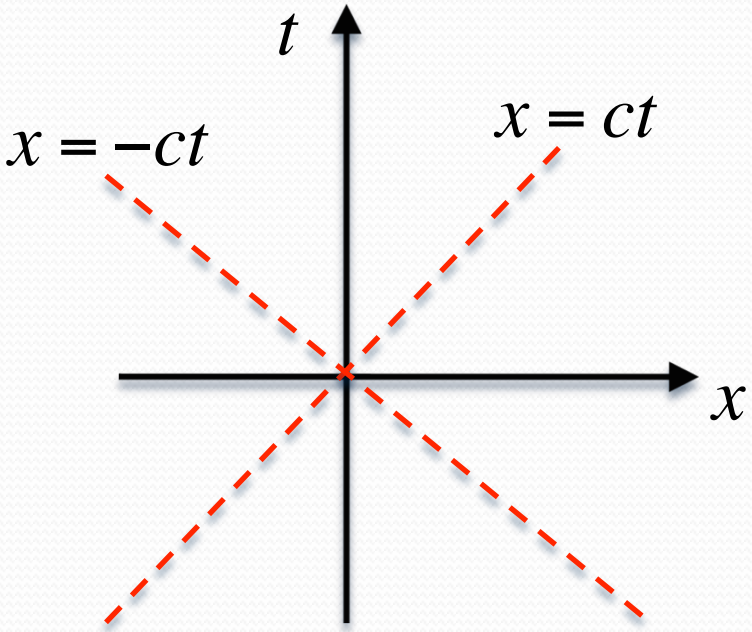
$$c = -\frac{dx}{dt}$$

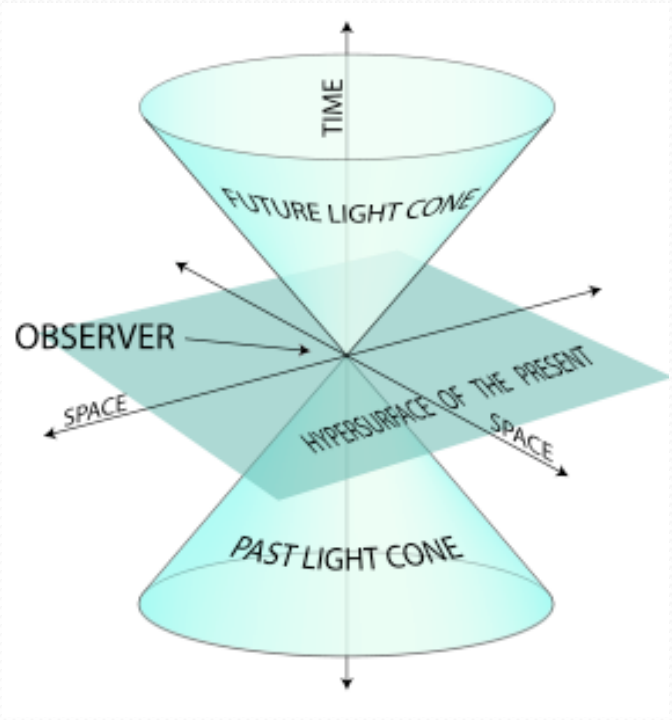
$$x(t) = c(t_0 - t)$$

EL ESPACIOTIEMPO DE LA RELATIVIDAD ESPECIAL

$$ds^2 \equiv dt^2 - (dx^2 + dy^2 + dz^2)$$

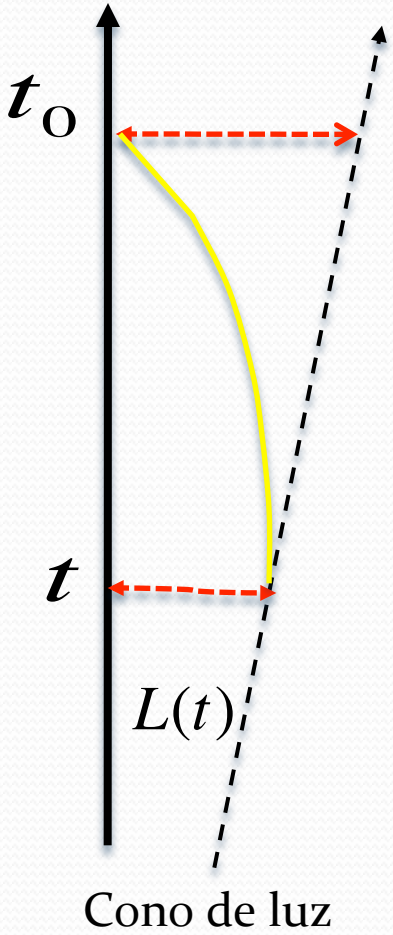
$$ds = 0 \Rightarrow \frac{dx}{dt} = \pm c$$





PROPAGACIÓN DE LUZ EN UN UNIVERSO EN EXPANSIÓN

$$c = -\frac{a(t)dx}{dt} \Rightarrow dx = -c\frac{dt}{a(t)}$$



Cono de luz

Distancia comóvil de los fotones

$$x(t) = c\int_t^{t_0} \frac{dt}{a(t)}$$

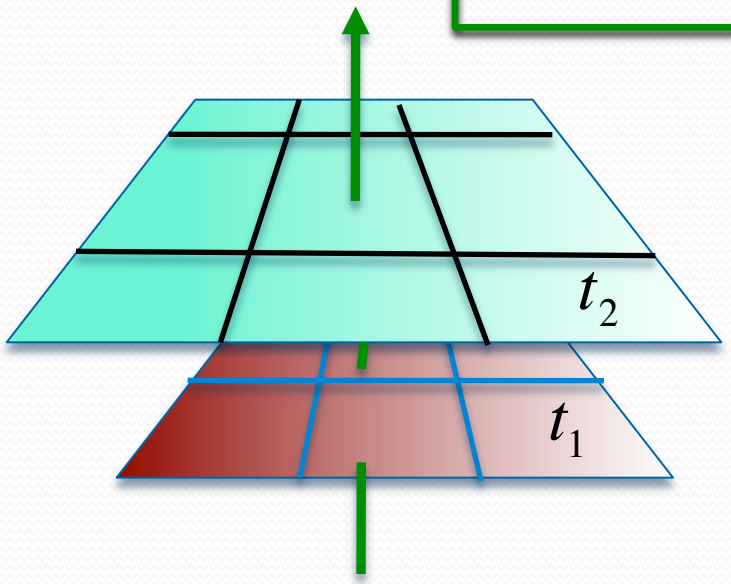
Distancia real de los fotones a nosotros :

$$L(t) = a(t)x(t)$$

$$L(t) = ca(t)\int_t^{t_0} \frac{dt}{a(t)}$$

EL ESPACIOTIEMPO COSMOLÓGICO : Métrica de Robertson-Walker k = o

$$ds^2 \equiv dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$



$$t = y = z = const.$$

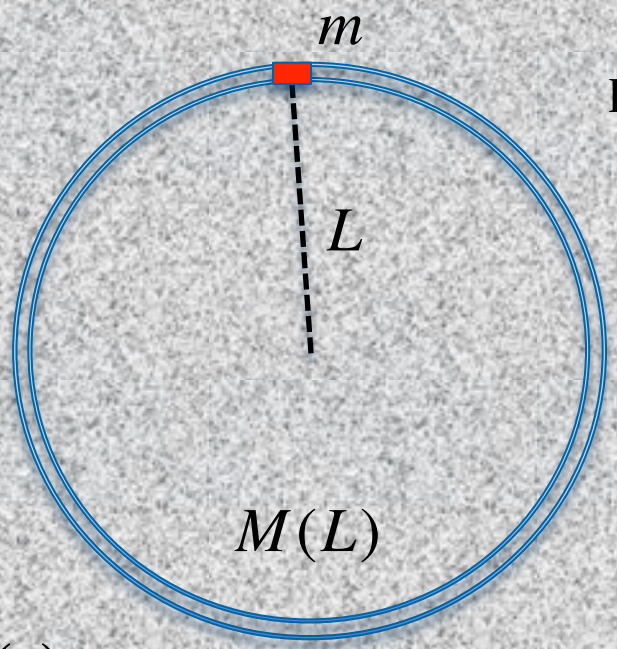
Distancias $$L = \int ds = a(t)\int dx = a(t)x_1$$

Propagación de la luz $$ds = 0$$

$$x = \pm c\int_t^{t_0} \frac{dt}{a(t)}$$

$$L(t) = ca(t)\int_t^{t_0} \frac{dt}{a(t)}$$

DINÁMICA DEL UNIVERSO: QUIÉN GOBIERNA A R(t)?
La ecuación de Friedman



La Energía es cero!!

$$\frac{1}{2}mV^2 - \frac{GM(L)m}{L} = 0$$

$$L = a(t)r_1 \qquad V = \dot{a}(t)r_1$$

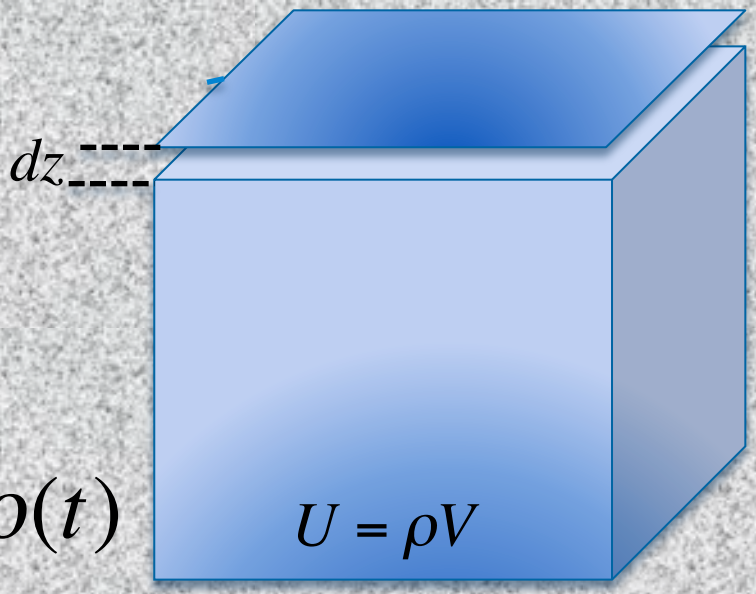
$$M(L) = \frac{4}{3}\pi(ar_1)^3\rho(t)$$

$$\frac{1}{2}r_1^2\dot{a}^2 - \frac{4\pi G}{3a(t)r_1}(ar_1)^3\rho(t) = 0$$

Ec. Friedman

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_T$$

QUIÉN GOBIERNA A LA DENSIDAD?



$$dU = -Fdz = -pdxdydz$$

$$U = \rho V$$

1 Ley Termodinámica

$$d(\rho V) = -pdV$$

$$Vd\rho = -PdV - \rho dV$$

$$\frac{d\rho}{dt} = -(p+\rho)\frac{dV}{Vdt}$$

$$V \sim a^3 \quad dV \sim 3a^2 da$$

$$\frac{dV}{V} = 3\frac{da}{a}$$

Ecuación de balance de energía

$$\dot{\rho} = -3\frac{\dot{a}}{a}(p+\rho)$$

LAS ECUACIONES DE LA COSMOLOGÍA

Ecuación Friedman

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_T$$

Ecuación Balance

$$\dot{\rho} = -3\frac{\dot{a}}{a}(p_T + \rho_T)$$

Ecuación Estado

$$p = p(\rho)$$

$$z + 1 = a^{-1}(t)$$

$$a = a(t) \quad \rightarrow \quad z = z(t)$$

$$a = a(z) \quad \rightarrow \quad t = t(z)$$

$$L(t) = ca(t)\int_t^{t_0} \frac{dt}{a(t)}$$

$$V = HL$$

EJEMPLOS DE MODELO COSMOLÓGICO –I-

Universo de materia: Modelo de Einstein –de Sitter

$p = 0$
Polvo

$$\Rightarrow \quad \dot{\rho} = 3\frac{\dot{a}}{a}(p_T + \rho_T) \quad \Rightarrow \quad \rho = \frac{A}{a^3}$$

Ec. Balance

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

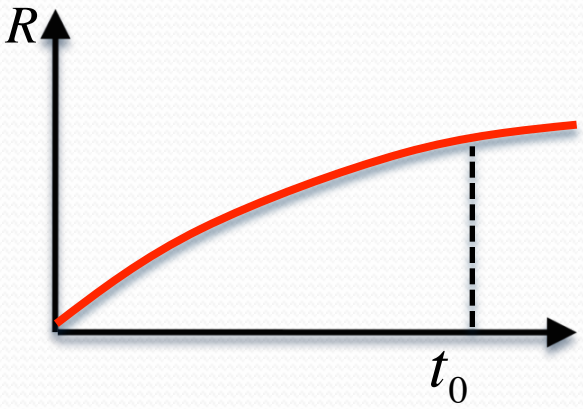
Ec. Friedman

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{B}{a^3} \quad \Rightarrow \quad a \sim t^{2/3} \quad \Rightarrow \quad a = (t/t_0)^{2/3}$$

$$H = \frac{2}{3t}$$

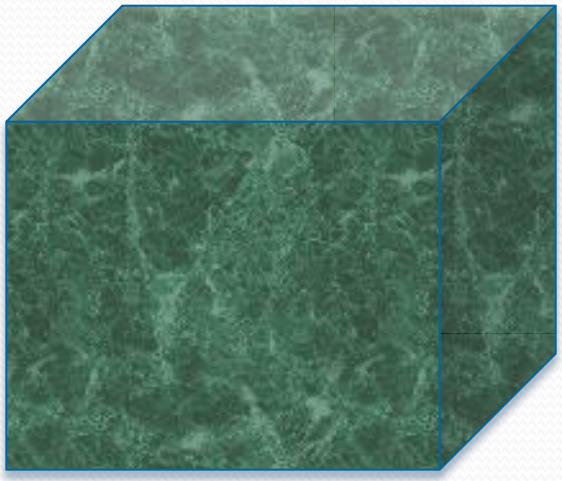
$$t_0 = \frac{2}{3H_0} \qquad q = \frac{1}{2}$$

$$z + 1 = \left(\frac{t_0}{t}\right)^{2/3}$$



$$L_{H_0} = \frac{3ct_0}{2} \approx 10^{10}\ años luz$$

ECUACIÓN DE ESTADO DE LA RADIACIÓN



Gas de fotones

Energía de un fotón $E_\gamma = h\upsilon$

$$U_{rad} = Nh\upsilon = \frac{Nhc}{\lambda}$$

$$\lambda = \frac{2L}{n} \qquad L = V^{1/3}$$

$$\Rightarrow \quad U_{rad} = BV^{-1/3}$$

$$dU = -Fdz = -pdxdydz$$

$$p = -\frac{\partial U}{\partial V}$$

$$p_{rad} = -\frac{\partial U_{rad}}{\partial V} = \frac{1}{3}\frac{BV^{-1/3}}{V}$$

$$\boxed{p_{rad} = \frac{\rho_{rad}}{3}}$$

EJEMPLO DE MODELO COSMOLÓGICO-II

Universo de radiación (Tolman)

$$p_r = \frac{\rho_r}{3}$$

Ec. de estado

$$\Rightarrow \quad \dot{\rho}_r = -4\frac{\dot{a}}{a}\rho_r \quad \Rightarrow \quad \rho_r = \frac{A}{a^4}$$

Ec. de balance

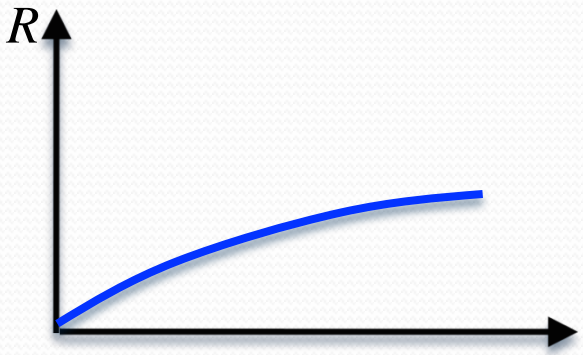
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{B}{a^4}$$

Ec. de Friedman

$$\Rightarrow \quad a \sim t^{1/2} \quad \Rightarrow \quad a = (t/t_0)^{1/2}$$

$$q = -1 \qquad H = \frac{1}{2t}$$

$$t_0 = \frac{1}{2H_0} \qquad z+1 = \left(\frac{t_0}{t}\right)^{1/2}$$



EJEMPLO DE MODELO COSMOLÓGICO -III

Universo de "vacío": Modelo de de-Sitter

$$\dot{\rho}_v = 3\frac{\dot{a}}{a}(p_v + \rho_v)$$

Ecuación de estado del vacío

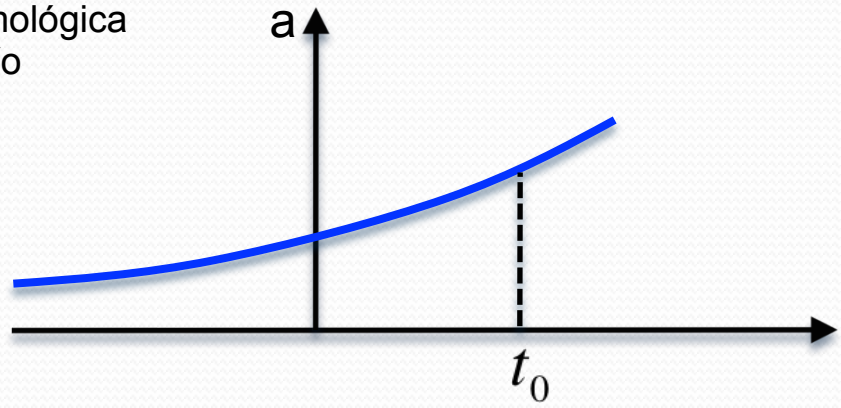
$$\frac{d\rho_v}{dt} = 0 \Rightarrow \boxed{p_v = -\rho_v}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3} \quad \Rightarrow \quad \boxed{a(t) = e^{\sqrt{\Lambda/3}t}}$$

$$\Lambda \equiv 8\pi G\rho_v$$

Constante Cosmológica
Energía del vacío
Energía oscura

$$H = \sqrt{\Lambda/3} \qquad q = -1$$



$$L_{H_0} = c\sqrt{3/\Lambda}$$

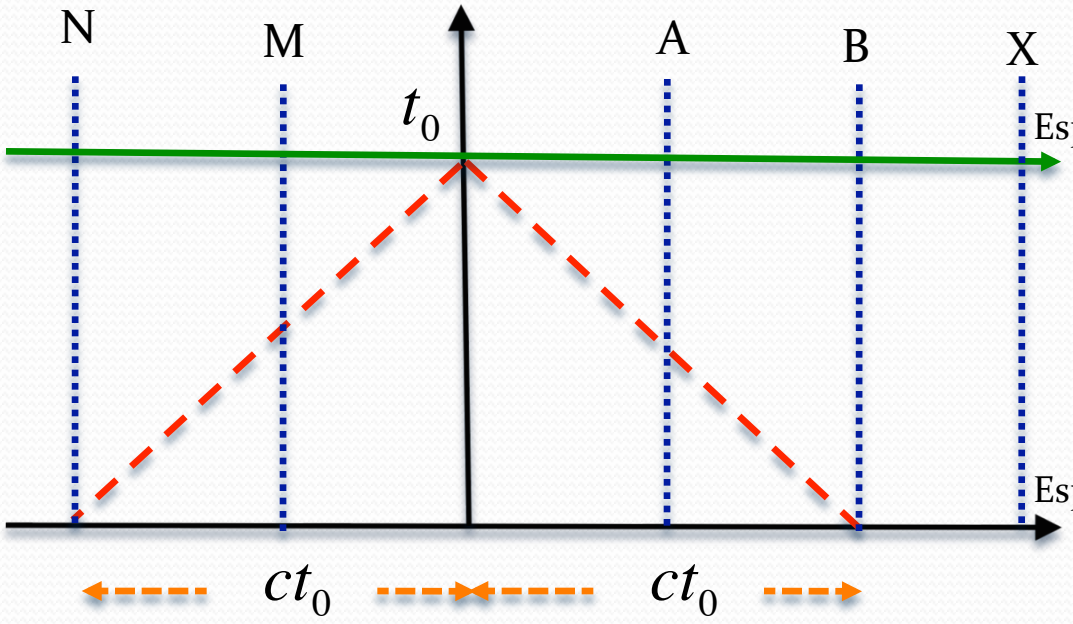
No hay big bang

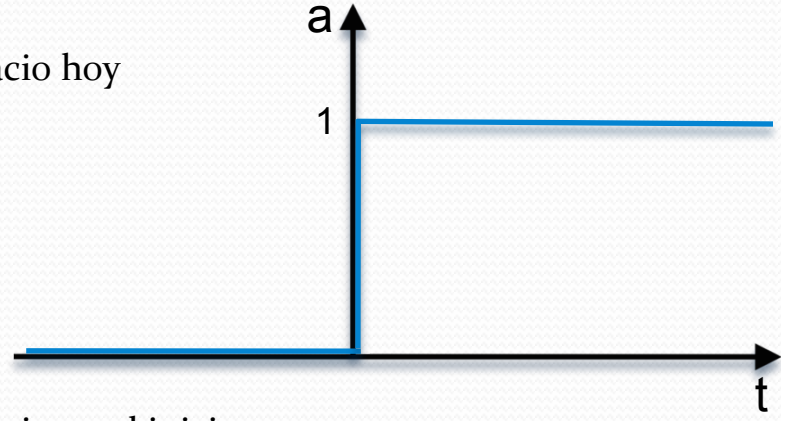
TRES MODELOS COSMOLOGICOS
RESUMEN

Nombre	Constituyente	Ec. estado	Fac. Escala a(t)	H	q
Einstein – de Sitter	Polvo	$p = 0$	$t^{2/3}$	$\frac{2}{3t}$	$\frac{1}{2}$
Tolman	Radiación	$p = \rho / 3$	$t^{1/2}$	$\frac{1}{2t}$	1
de - Sitter	Vacío	$\rho = const$	$\exp \alpha t$	$\sqrt{\Lambda} / 3$	-1

HORIZONTES

Horizonte de partículas. Un espacio euclidiano abruptamente creado





Def: Horizonte de partícula

$$L_{horiz}(t_0) = c\int_{t_i}^{t_0} \frac{dt}{a(t)}$$

Ejemplo: Einstein de-Sitter

$$L_{horiz}(t_0) = ct_0^{2/3}\int_{t_i}^{t_0} \frac{dt}{t^{2/3}} = 3ct_0$$

HORIZONTES

Velocidad a la que aumenta la distancia al horizonte

En un tiempo t $$L_{horiz}(t) = ca(t)\int_0^t \frac{dt}{a(t)}$$

$$\frac{dL_{horiz}(t)}{dt} = c\dot{a}(t)\int_0^t \frac{dt}{a(t)} + ca(t)\frac{1}{a(t)}$$

$$\boxed{\frac{dL_{horiz}}{dt} = H(t)L_{horiz} + c}$$

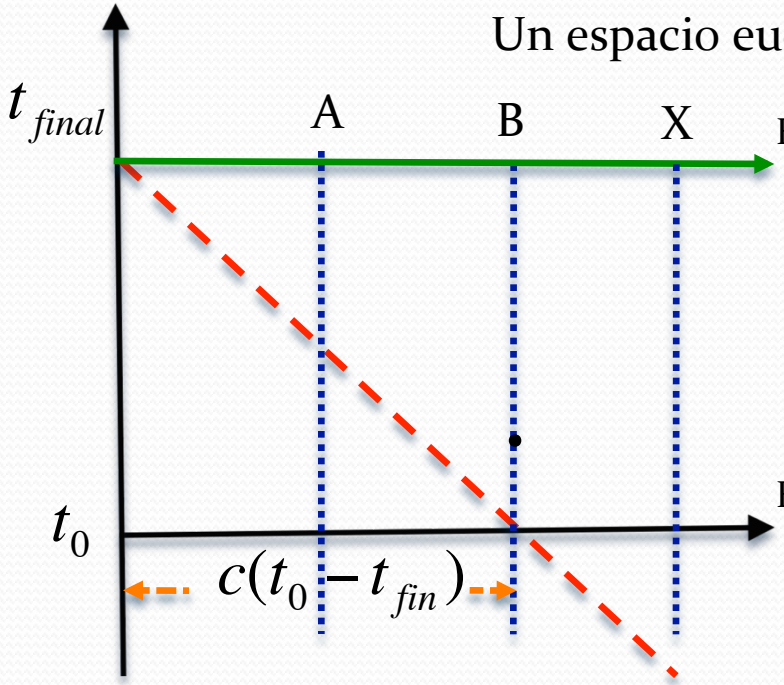
Ejemplo: Modelo de Einstein-deSitter $$\frac{dL_{horiz}}{dt} = \frac{2}{3t}(3ct) + c = 3c$$

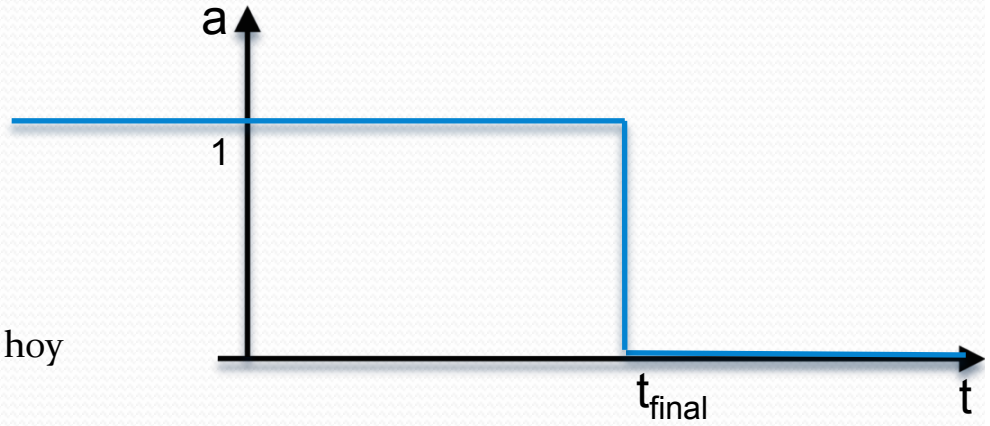
Puede verse que $$\frac{dN_{horiz}}{dt} > 0$$

HORIZONTES

Horizonte de eventos:

Un espacio euclidiano abruptamente finalizado





Def: Horizonte de eventos

$$L_{event}(t_0) = c\int_{t_0}^{t_f} \frac{dt}{a(t)}$$

Ejemplo: Einstein de-Sitter

$$L_{event}(t_0) = ct_0^{2/3}\int_{t_0}^{\infty} \frac{dt}{t^{2/3}} = \infty$$