EL UNIVERSO AL ALCANCE DEL CÁLCULO

PARTE IV
APROXIMACIÓN AL UNIVERSO REAL

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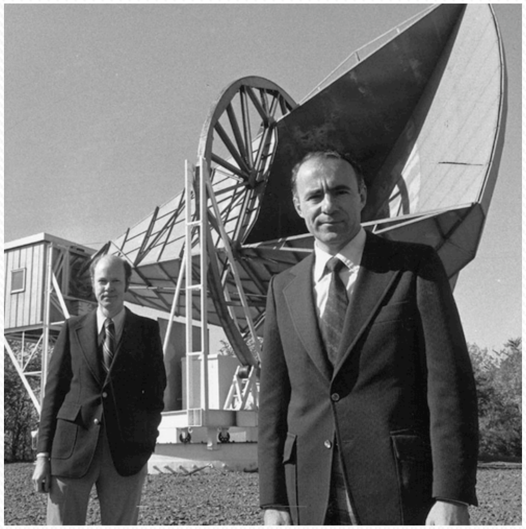
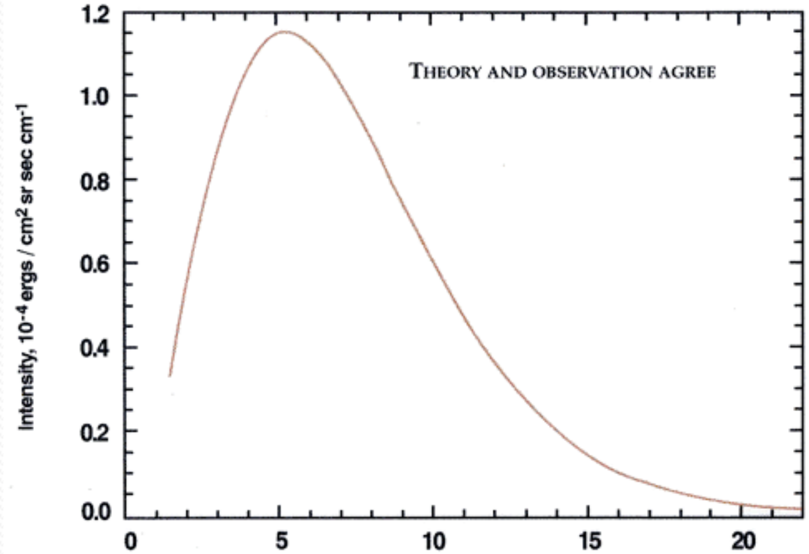
APROXIMACIÓN AL UNIVERSO REAL

La radiación cósmica de fondo

Penzias &Wilson
Pebbles & Dicke
Gamow
1965





$T_0 = 2,73^o\,K$

Ley de Stephan-Boltzman $\rho_r = aT^4$ $\rho_r \sim R^{-4}$

$\rho_{r,0} = 4,6 \times 10^{-34}\, g.cm^{-3}$

$\Omega_r = 5 \times 10^{-5}$

$\rho_T = 9 \times 10^{-30}\, g.cm^{-3}$

$T \sim a^{-1}$

$a^{-1} = z + 1$

$T(z) = T_0(1+z)$

$T_0 = 2,73^o\,K$

$T(BB) = \infty$

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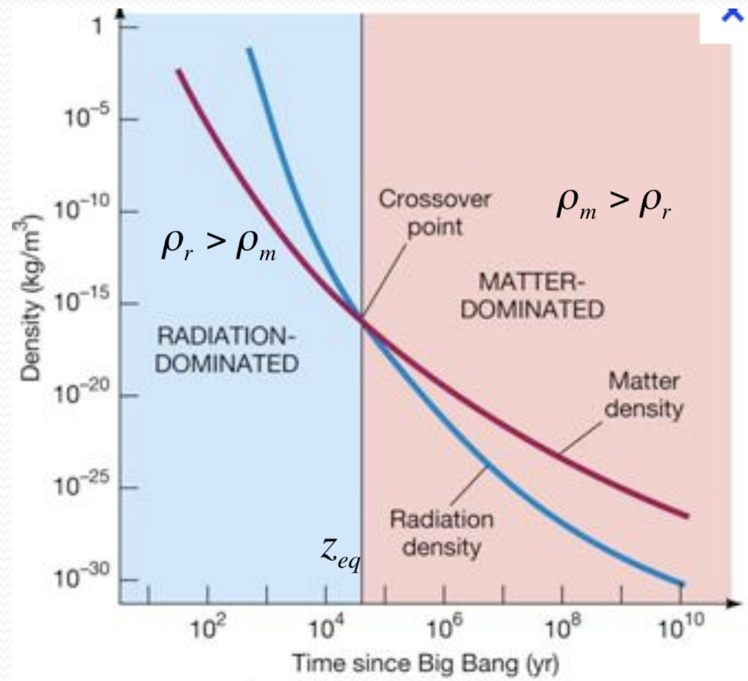
La radiación cósmica de fondo

$$\rho_r = \rho_{r,0} a^{-4}$$
$$\rho_m = \rho_{m,0} a^{-3}$$

$$\Rightarrow \rho_r(z_{eq}) = \rho_m(z_{eq})$$

$$\rho_r = \rho_{r,0}(1+z)^4$$
$$\rho_m = \rho_{m,0}(1+z)^3$$

$$\Rightarrow \frac{\rho_r}{\rho_m} = \frac{\rho_{r,0}}{\rho_{m,0}}(1+z) = 1$$



Rotación de galaxias
Dinámica de cúmulos globulares
Lentes gravitacionales

$$\Rightarrow \rho_{m,0} = 2,7 \times 10^{-30}\, g.cm^{-3}$$

$$\frac{\rho_{m,0}}{\rho_{T,0}} \equiv \Omega_{m,0} = 0,27$$

$$1 + z_{eq} = \frac{\rho_{m,0}}{\rho_{r,0}} = \frac{\Omega_{m,0}}{\rho_{r,0}} = \frac{0,3}{5 \times 10^{-5}}$$

$$\Downarrow$$

$$z_{eq} = 6000$$

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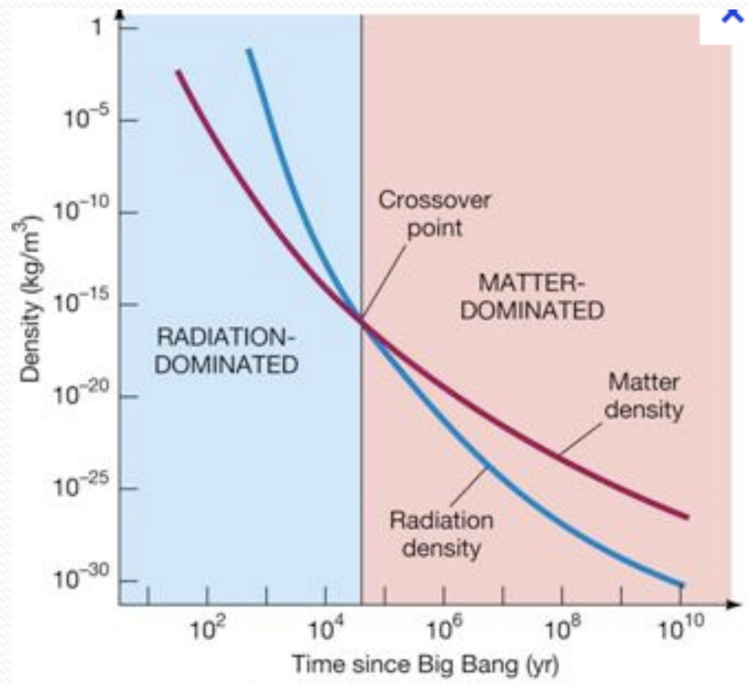
La radiación cósmica de fondo

Qué edad tenía el universo cuando $z_{eq} = 6000$?

Einstein-deSitter $a(t) \sim t^{2/3} \Rightarrow \frac{t}{t_0} = (z+1)^{3/2}$

$$t_{eq} = t_0 (6000)^{-3/2} = \frac{t_0}{5\times 10^5}$$

$t_0 \approx 14\times 10^9 \, años$ ⇨ $t_{eq} \approx 5\times 10^4 \, años$



De qué tamaño era el universo cuando $z_{eq} = 6000$?

$$L_{horiz}(t_{eq}) = c a_{eq} \int_0^{t_{eq}} \frac{dt}{a(t)}$$

Usando el modelo de radiación $a \sim t^{1/2}$ $a(t) = a_{eq} \frac{t^{1/2}}{t_{eq}^{1/2}}$

$$L_{horiz}(t_{eq}) = 2ct_{eq}$$

$$L_{horiz}(t_{eq}) = c\times 10^5 \, años$$

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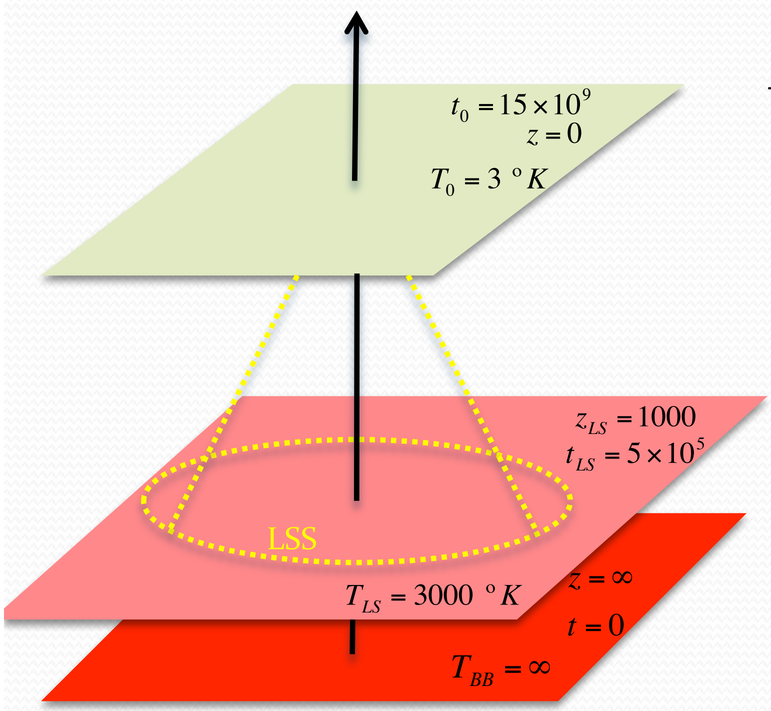
La superficie de último scattering

La superficie de "last scattering"



Cuando T = 3000ºK se forman átomos de H
Como To = 3ºK, z = 1000

$$\frac{t_{LS}}{t_0} = (z_{LS}+1)^{-3/2} \Rightarrow t_{LS} = 5\times 10^5\, años$$

Horizonte en ese instante (suponiendo E-d S)

$$L_{horiz}(t_{LS}) = 3ct_0(z_{LS}+1)^{-3/2}$$

Esa distancia hoy es $D_0(t_{LS}) = 3ct_0(z_{LS}+1)^{-3/2}\times(z_{LS}+1)$

Distancia actual de la superficie LS

$$L_0(z_{LS}) = 2L_{H_0}(1-1/\sqrt{1000}) = 2L_{H_0}(20/30)$$

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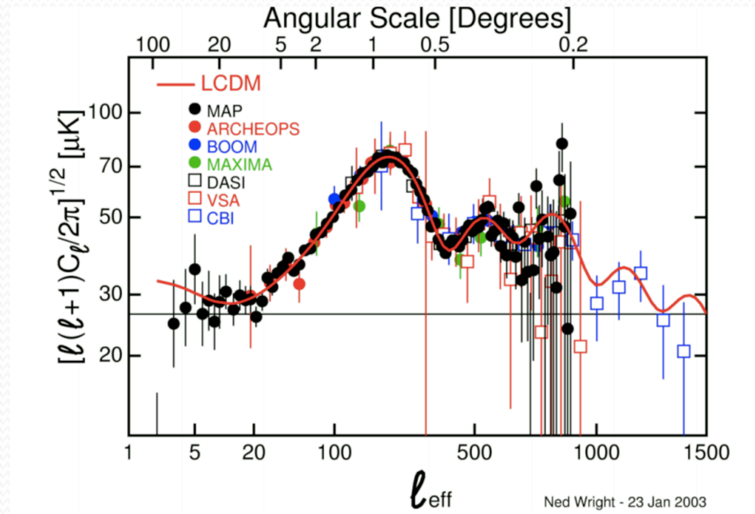
Anisotropía de la temperatura de la CBR

Tamaño del horizonte sónico y la distancia de la superficie de último scattering depende de los parámetros cosmológicos como la densidad total, la densidad de bariones, la curvatura (energía), densidad de la materia…



$H_o = 71$, $\Omega_\Lambda = 0.73$, $\Omega_b h^2 = 0.0224$, $\Omega_m h^2 = 0.135$, $\Omega_{tot} = 1$

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Un universo de materia y de vacío

Un universo sólo de materia?

$$a(t) \sim t^{2/3} \quad \Rightarrow \quad t_0^{EdS} = \frac{2}{3H_0} \quad \Rightarrow \quad t_0^{EdS} = 9{,}2 \times 10^9 \, años$$

$$H_0^{-1} = 13{,}8 \times 10^9 \, años$$

Incompatible con la edad el sol

⇩

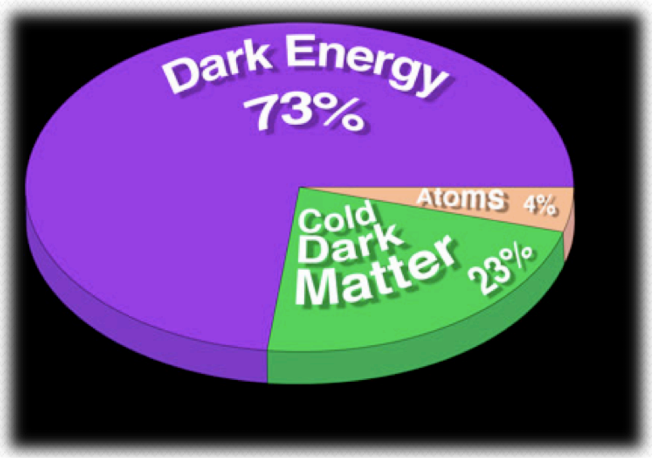
Hay algo más que Masas (materia) y radiación (fotones)

La contribución del vacío

$$\Omega_m + \Omega_r + \Omega_x = 1 \quad \Rightarrow \quad \Omega_x = \Omega_v \approx 0{,}73$$

$\Omega_m \to 0{,}27$ $\quad \Omega_r \to 10^{-5}$

Ecuación de estado $p_v = -\rho_v \Leftrightarrow \dot{\rho}_v = 0$



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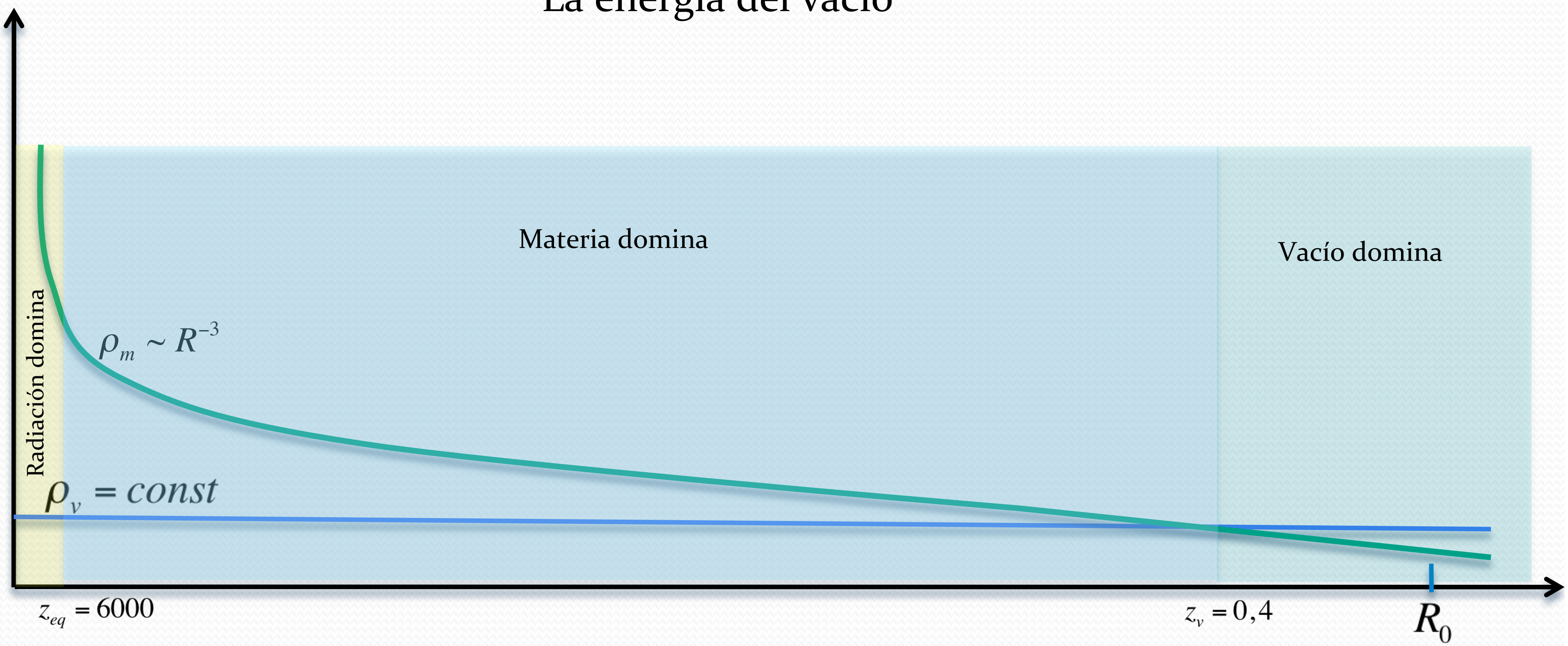
La energía del vacío



Cuándo se hicieron iguales las densidades de materia y energía del vacío?

$$\frac{\rho_m}{\rho_v} = \frac{\rho_{m,0}(1+z)^3}{\rho_v} = \frac{\Omega_m}{\Omega_v}(1+z)^3 \qquad \rho_m = \rho_v \quad \Rightarrow \frac{\Omega_m}{\Omega_v}(1+z_v)^3 = 1 \quad \Rightarrow \quad z_v = 0,4$$

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El factor de escala del universo

Evolución general de a(t)
Ecuación de Friedman

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho_m + \rho_v)$$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho_m + \rho_v)$$

$$\rho_m = \rho_{m,0} a^{-3}$$
$$\rho_v = \rho_v$$

$$\frac{\dot{a}^2}{a^2} = \frac{H_0^2}{\rho_T}(\rho_{m,0} a^{-3} + \rho_v)$$

$$\frac{\dot{a}^2}{a^2} = H_0^2(\Omega_{m,0} a^{-3} + \Omega_v)$$

$$dt = \frac{1}{H_0}\frac{da}{\sqrt{\Omega_m a^{-1} + \Omega_v a^2}}$$

$$H_0 t = \int_0^a \frac{da}{\sqrt{\Omega_m a^{-1} + \Omega_v a^2}}$$

$a << 1 \rightarrow$ Einstein-de Sitter

$a >> 1 \rightarrow$ De Sitter

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El factor de escala del universo

$$H_0 t = \int_0^a \frac{\sqrt{a}da}{\sqrt{\Omega_m + \Omega_v a^3}} = \frac{1}{\sqrt{\Omega_m}} \int_0^a \frac{\sqrt{a}da}{\sqrt{1 + (\Omega_v / \Omega_m) a^3}}$$

haciendo $x^2 = (\Omega_v / \Omega_m) a^3$

$$\frac{3}{2} H_0 \sqrt{\Omega_v} \; t = \int_0^x \frac{dx}{\sqrt{1+x^2}} = \sinh^{-1} x$$

$$A \equiv (\Omega_m / \Omega_v)^{1/3}$$

$$t_v = \frac{2}{3 H_0 \sqrt{\Omega_{v,0}}}$$

$$t_v = 10{,}77 \times 10^9 \, años$$

⇨

$$a(t) = A \sinh^{2/3}(t / t_v)$$

→

t_0

H

q

$z = z(t)$

$L(z)$

L_H

L_{horiz}

$V(z)$

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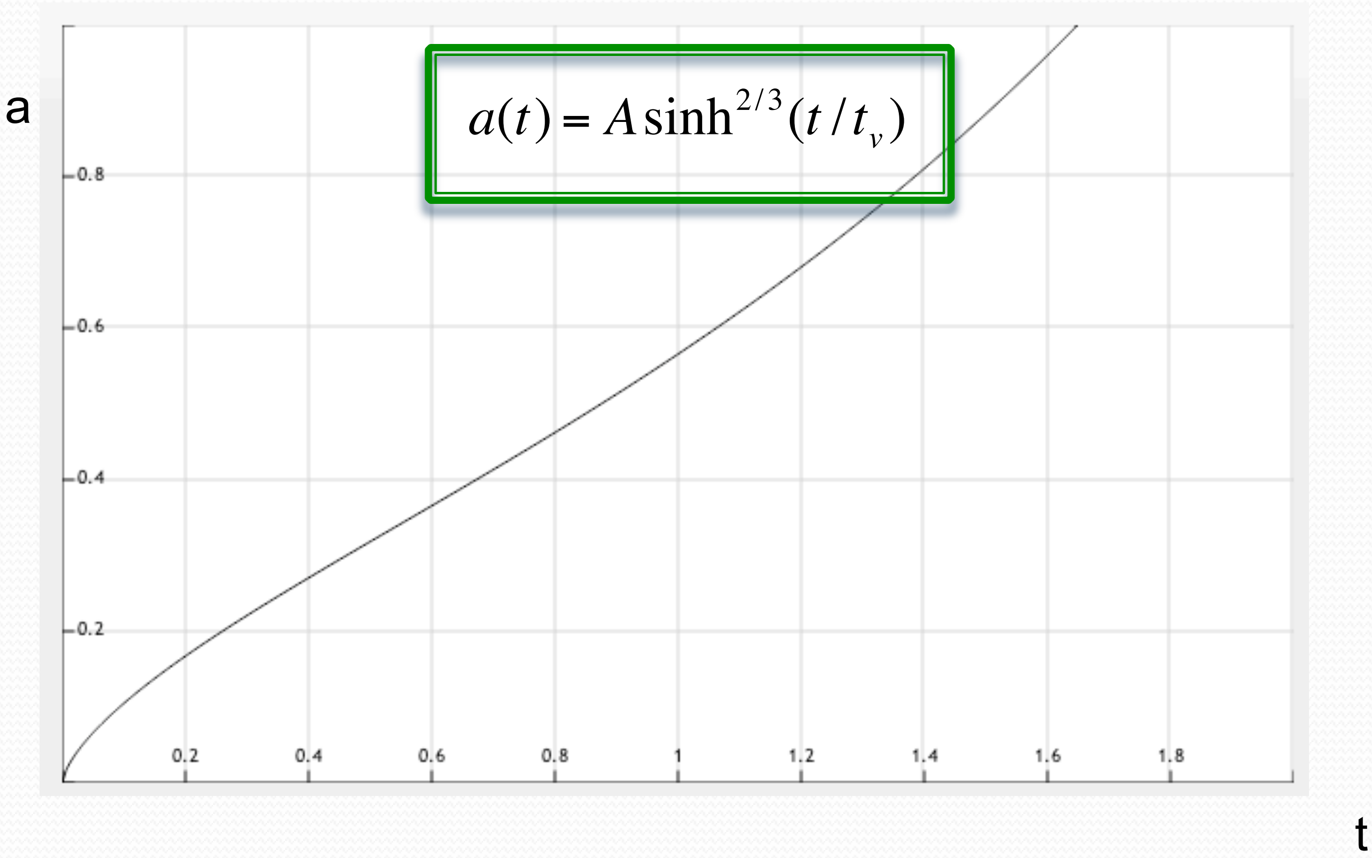
El factor de escala del universo

$$a(t) = A \sinh^{2/3}(t/t_v)$$



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La edad del universo

$$a(t) = A\sinh^{2/3}(t/t_v) \longrightarrow 1 = A\sinh^{2/3}(t_0/t_v)$$

$$\sinh(t_0/t_v) = B \qquad B \equiv A^{-3/2} = \sqrt{\frac{\Omega_v}{\Omega_m}}$$

$$\tanh(t_0/t_v) = \frac{B}{\sqrt{1+B^2}}$$

Sustituyendo B $\quad \tanh(t_0/t_v) = \sqrt{\Omega_v} \Rightarrow$

$$t_0 = t_v \operatorname{arctan}h\sqrt{\Omega_v}$$

$$\operatorname{arctan}hx = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$t_0 = \frac{2}{3H_0\sqrt{\Omega_v}} \times \frac{1}{2}\ln\left(\frac{1+\sqrt{\Omega_v}}{1-\sqrt{\Omega_v}}\right)$$

=9,2 x10⁹ años (primer factor) — =1,49 (segundo factor)

$$t_0 = 13{,}7\times10^9\,años$$

$$t_v = 10{,}77\times10^9\,años$$

$$\frac{t_0}{t_v} = 1{,}27$$

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La constante de Hubble H

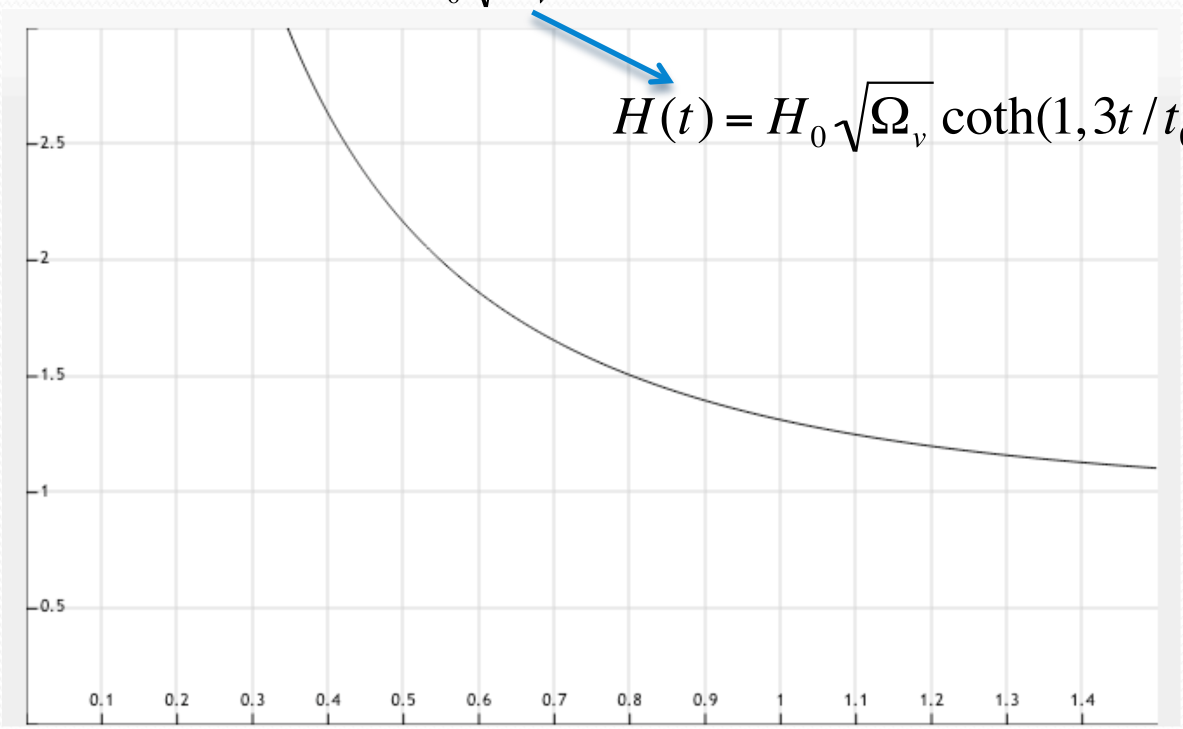
$$a(t) = A\sinh^{2/3}(t/t_v) \quad \Rightarrow \quad \dot{a}(t) = \frac{2}{3t_v}\frac{A\sinh^{2/3}(t/t_v)}{\sinh(t/t_v)}\cosh(t/t_v)$$

$$\boxed{H(t) = \frac{2}{3t_v}\coth(t/t_v)}$$

$$t_v = \frac{2}{3H_0\sqrt{\Omega_v}}$$

$$H(t) = H_0\sqrt{\Omega_v}\,\coth(1,3t/t_0)$$

$H(t)$



$\dfrac{t}{t_0}$

$$\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$x \to \infty, \quad \coth(x) \to 1$$

$$H(t) \to H_0\sqrt{\Omega_v}$$

$$L_H \to \frac{c}{H_0\sqrt{\Omega_v}}$$

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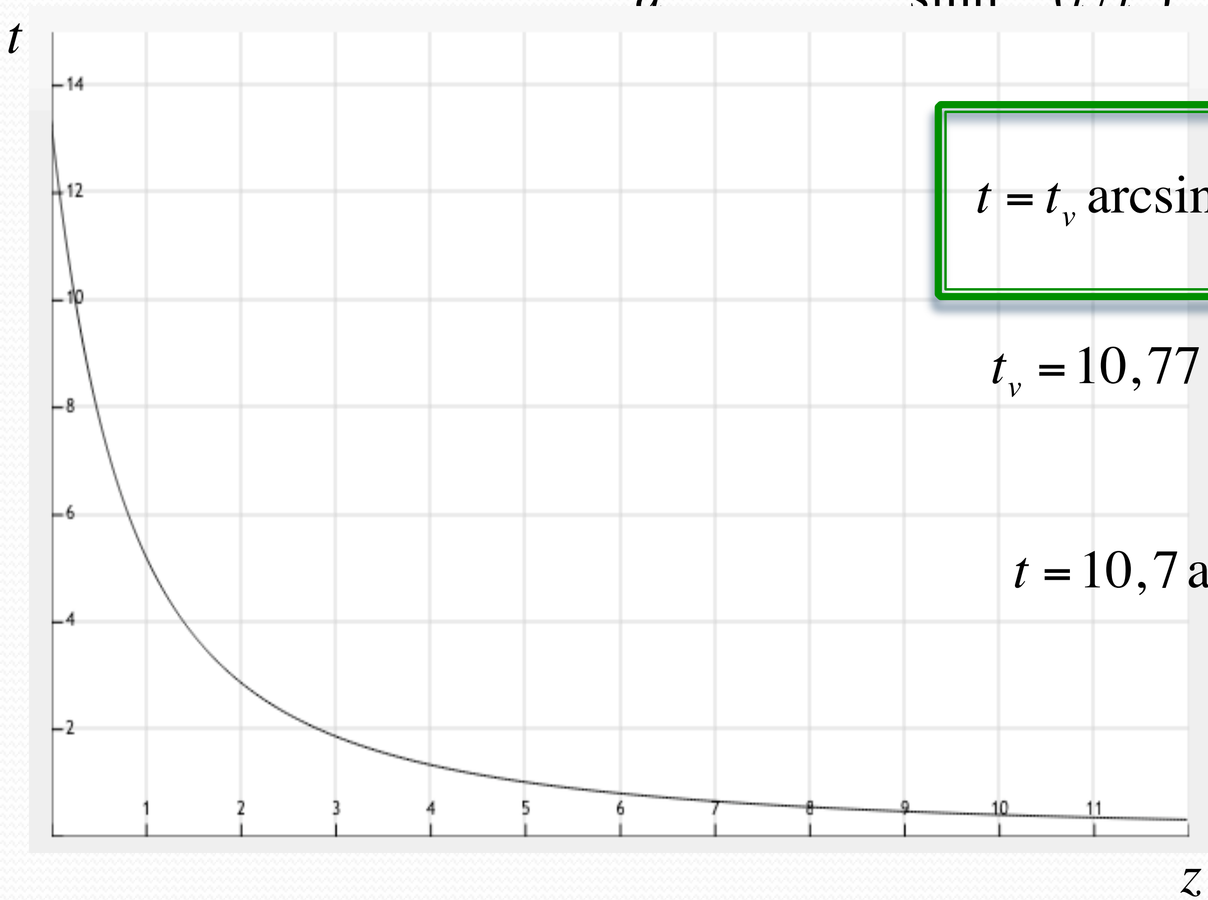
El corrimiento al rojo

$$a(t) = A \sinh^{2/3}(t/t_v) \qquad a(t_0) = A \sinh^{2/3}(t_0/t_v)$$

$$\frac{a(t_0)}{a} = z + 1 = \frac{\sinh^{2/3}(t_0/t_v)}{\sinh^{2/3}(t/t_v)}$$



$$t = t_v \operatorname{arcsin} h\left[\frac{\sinh(t_0/t_v)}{(z+1)^{3/2}}\right]$$

$$z = 0 \rightarrow t = t_0$$

$$z = \infty \rightarrow t = 0$$

$$t_v = 10{,}77 \times 10^9 \, años \qquad \frac{t_0}{t_v} = 1{,}27$$

$$t = 10{,}7 \operatorname{arcsin} h\left[\frac{2}{(z+1)^{3/2}}\right]$$

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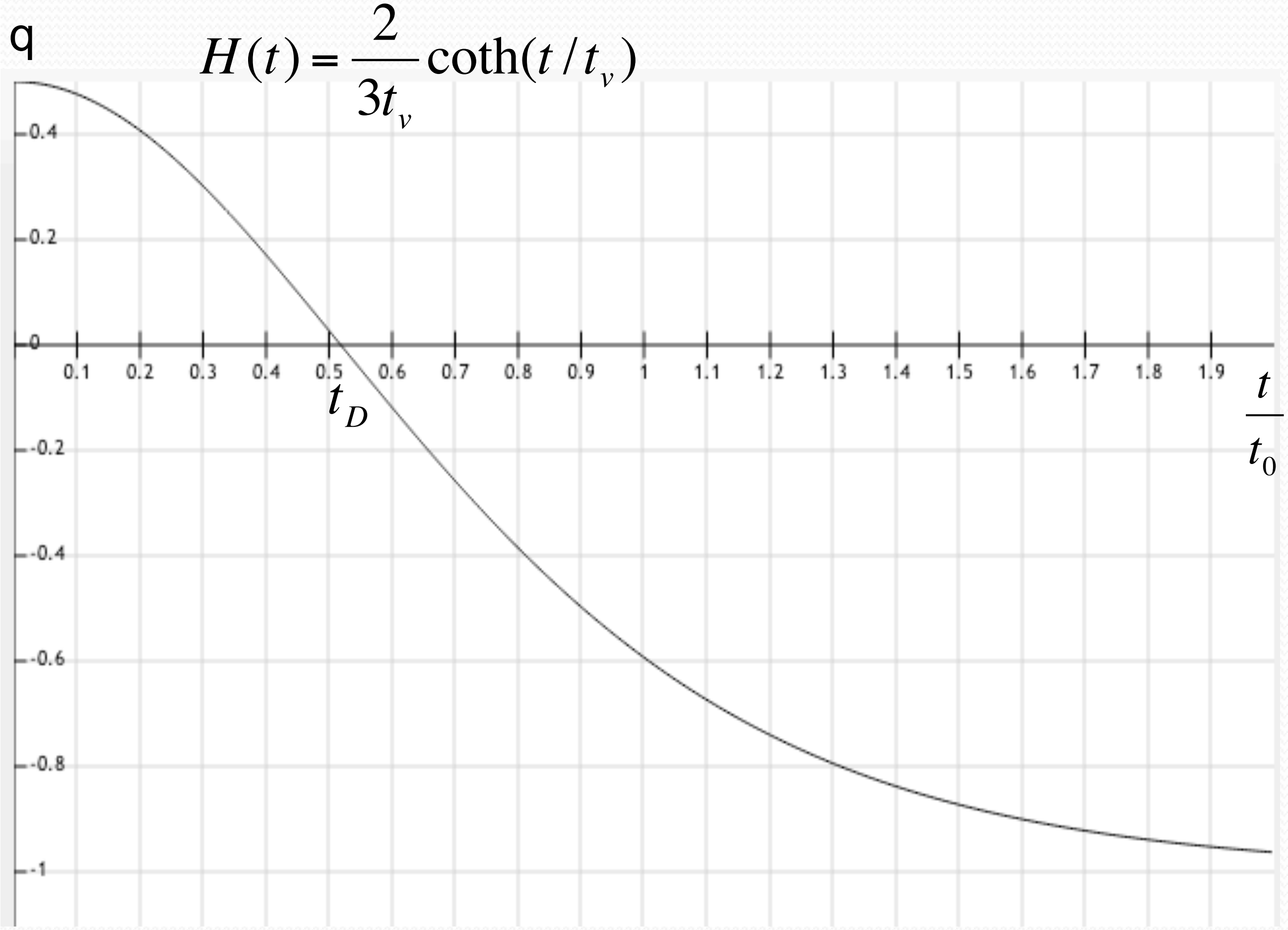
Parámetro de desaceleración

$$q \equiv -\frac{\ddot{a}}{aH^2}$$

$$a(t) = A\sinh^{2/3}(t/t_v)$$

$$\ddot{a}(t) = \frac{2}{3t_v} A\sinh^{2/3}(t/t_v)\left[1 - \frac{\coth^2(t/t_v)}{3}\right] \Rightarrow q(t) = \frac{1}{2}[1 - 3\tanh^2(t/t_v)]$$

$$H(t) = \frac{2}{3t_v}\coth(t/t_v)$$



$$q(t) = 0$$

$$\Downarrow$$

$$t_D = t_v \arctan h(1/\sqrt{3})$$

$$t_D = 7\times 10^9 \, años$$

$$\Downarrow$$ Ver problemas

$$z_D = \left(\frac{2\Omega_v}{1-\Omega_v}\right)^{1/3} - 1$$

$$z_D = 0,75$$

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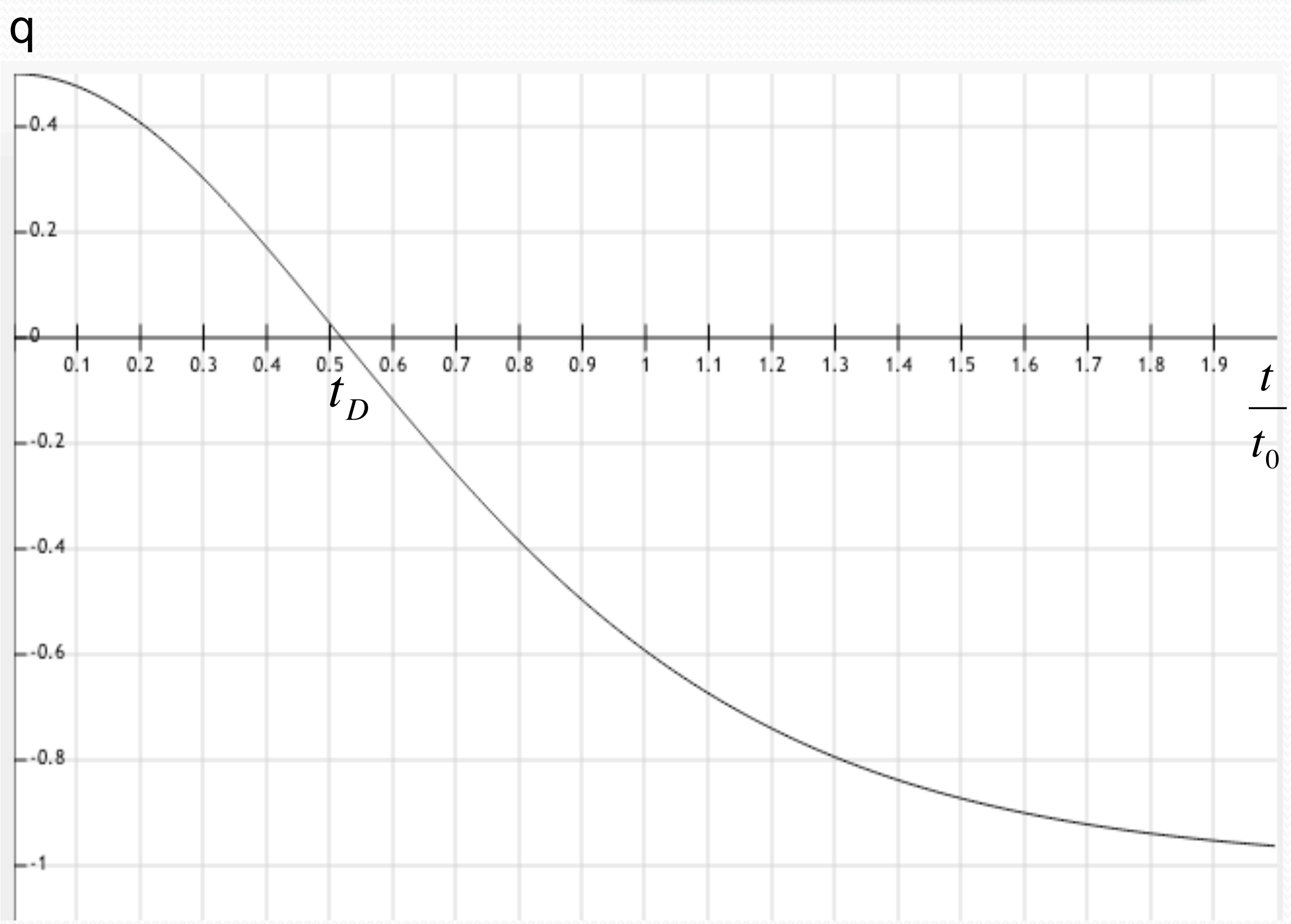
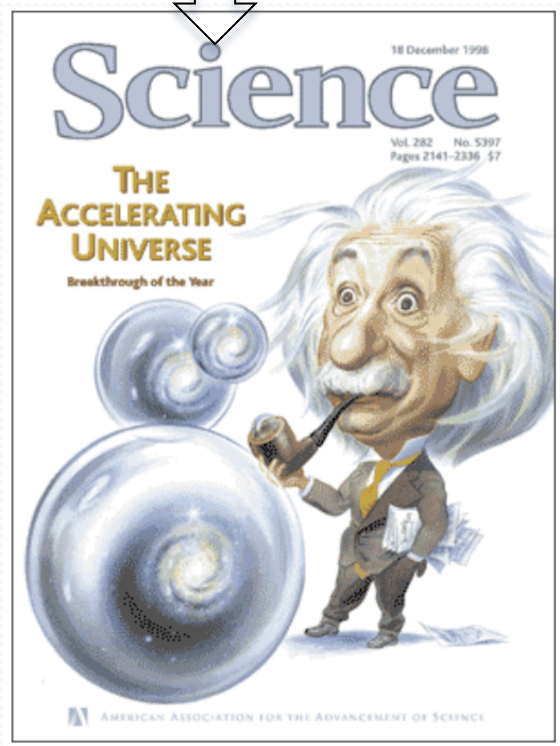
Parámetro de desaceleración hoy

$$q(t) = \frac{1}{2}[1 - 3\tanh^2(t/t_v)]$$

$$q_0 = \frac{1}{2}[1 - 3\tanh^2(1,27)]$$

$$q_0 = -0,59$$





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Evidencias de la aceleración: Supernovas

$$q_0 = -0,61$$

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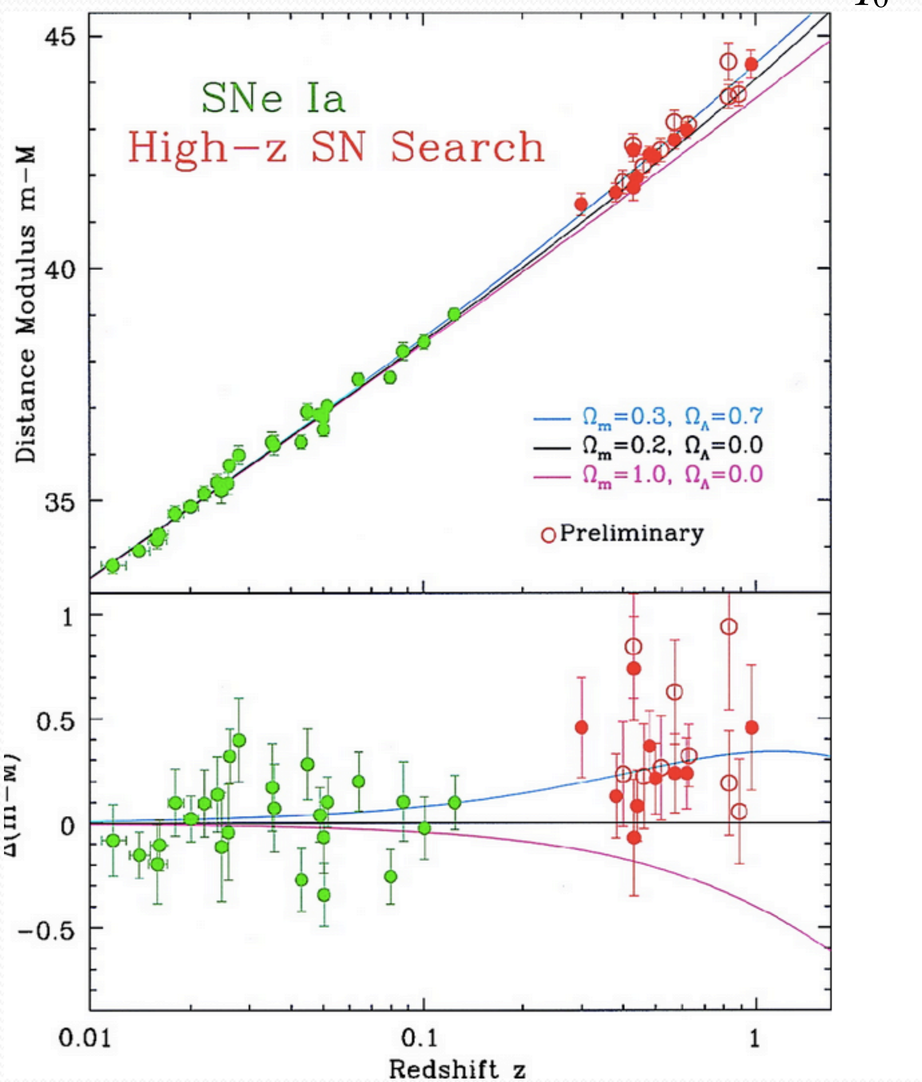
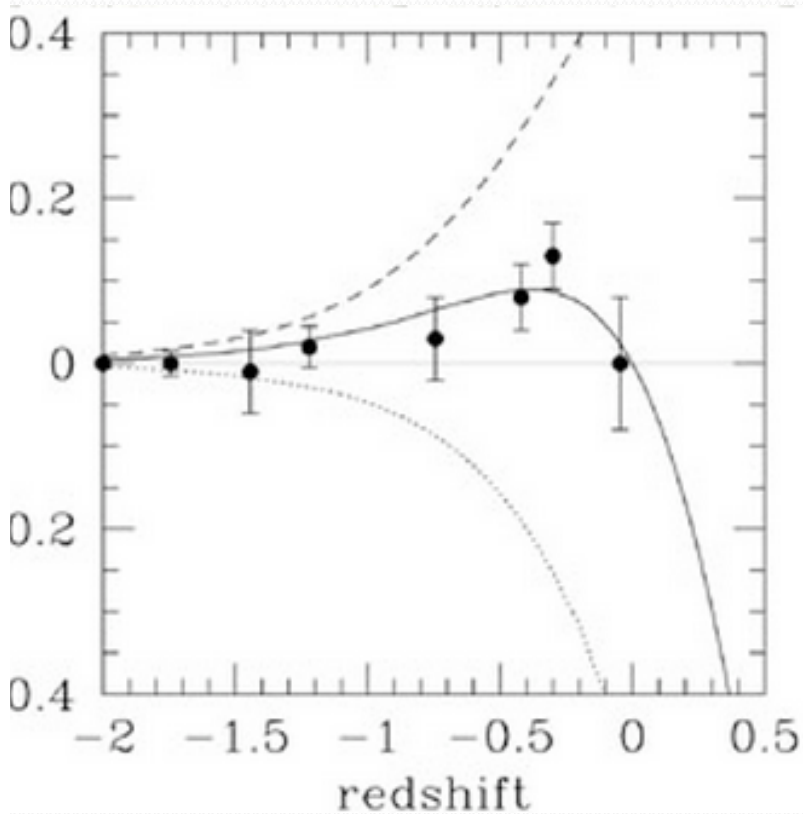
Evidencias de la aceleración

$$q_0 = -0,61$$





El parámetro densidad del vacío

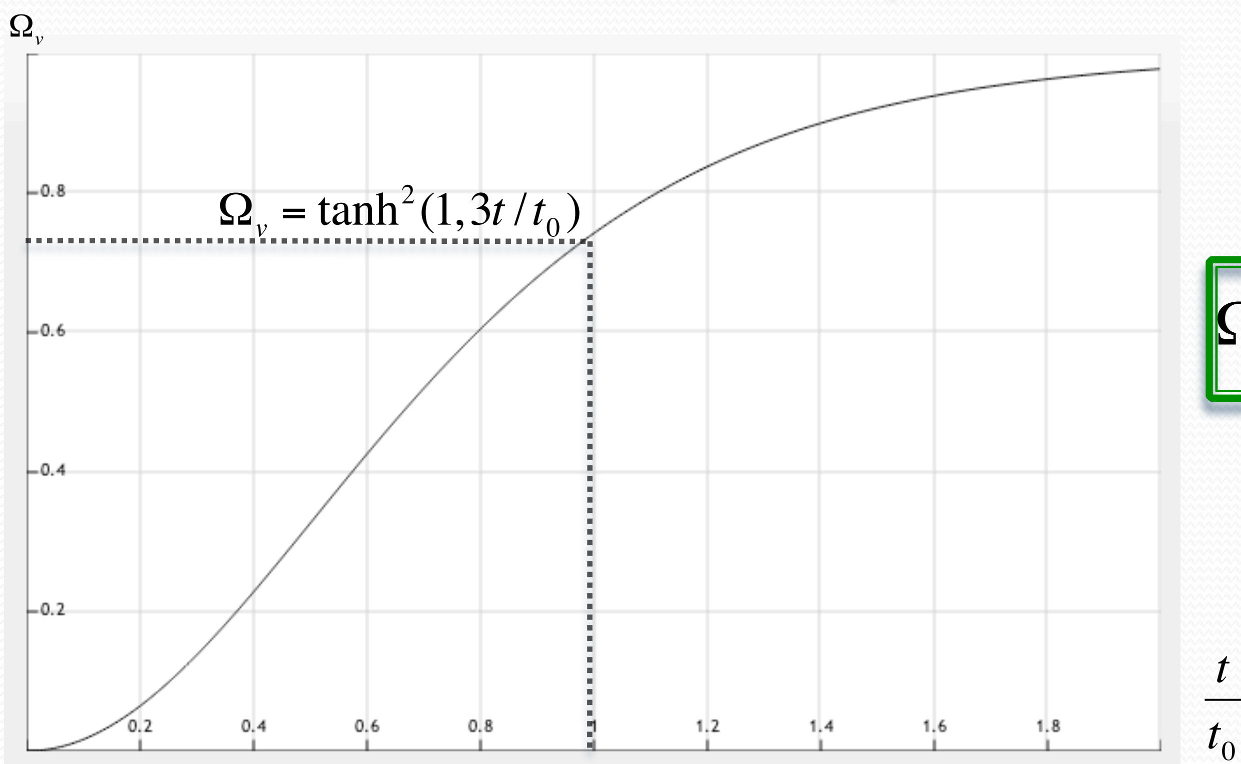
Friedman: $\rho_T = \dfrac{3H^2}{8\pi G}$ $\qquad H^2 = \dfrac{4}{9t_v^2}\tanh^{-2}(t/t_v)$ $\qquad t_v^2 = \dfrac{4}{9H_0^2\Omega_v}$

$$\rho_T = \frac{3H_0^{\ 2}}{8\pi G}\Omega_v \tanh^{-2}(t/t_v) \quad \Rightarrow \quad \rho_T = \rho_v \tanh^{-2}(t/t_v)$$

$$\Omega_v = \frac{\rho_v}{\rho_T}$$

$$\Downarrow$$

$$\boxed{\Omega_v(t) = \tanh^2(t/t_v)}$$



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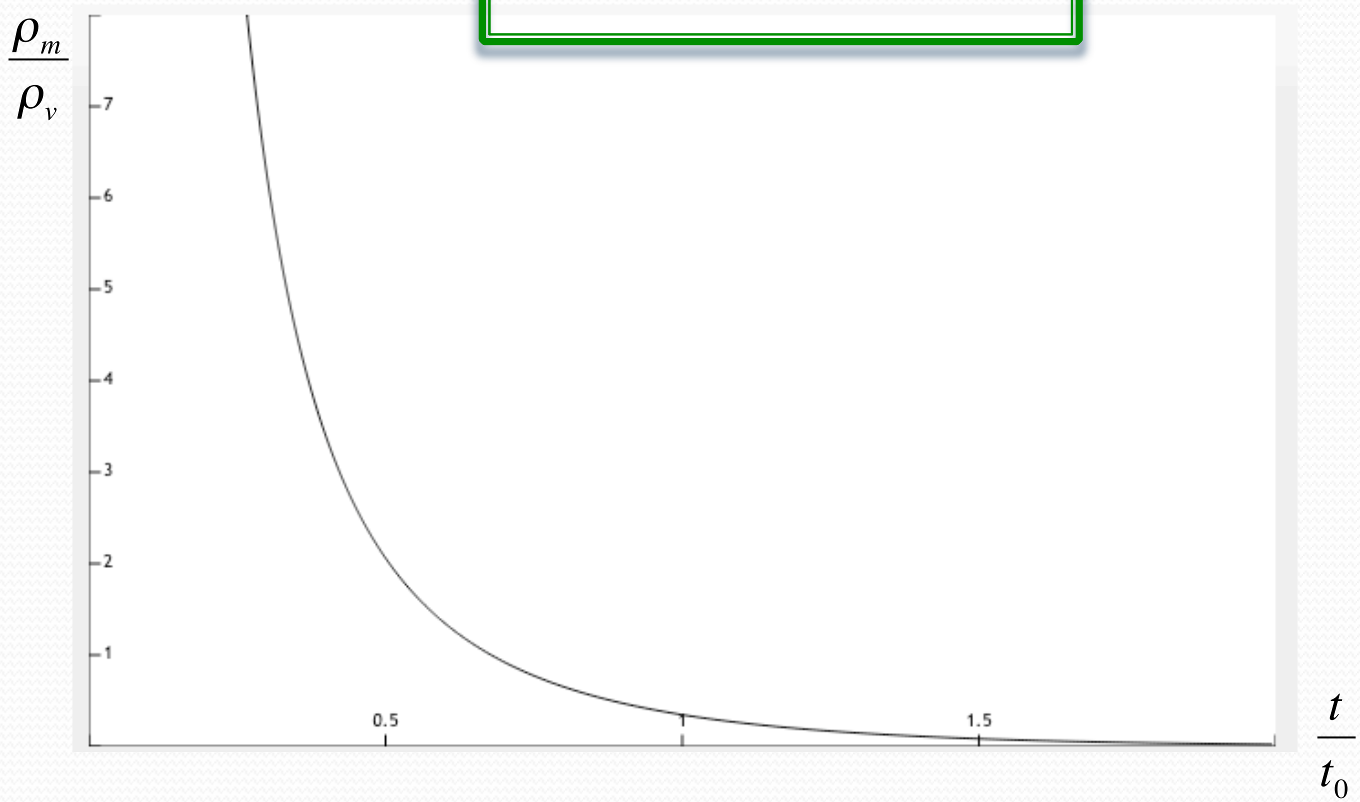
Las proporciones de materia y energía

Densidad de materia

$$\rho_m = \rho_{m,0} a^{-3} \quad \text{pero} \quad a^3 = \left(\frac{\Omega_{m,0}}{\Omega_{v,0}}\right) \sinh^2(t/t_v)$$

$$\rho_m(t) = \rho_v \sinh^{-2}(t/t_v)$$



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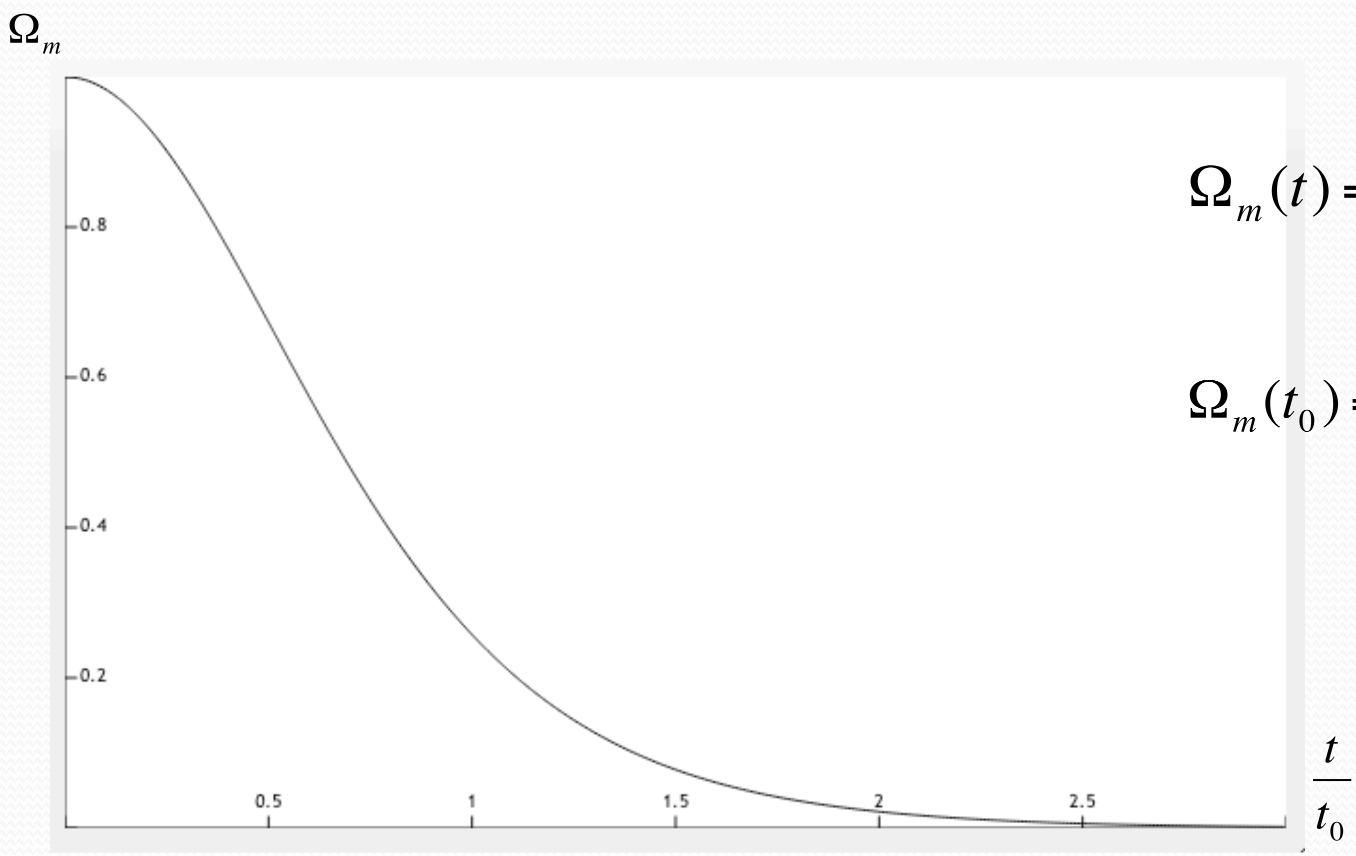
Proporción de materia y energía

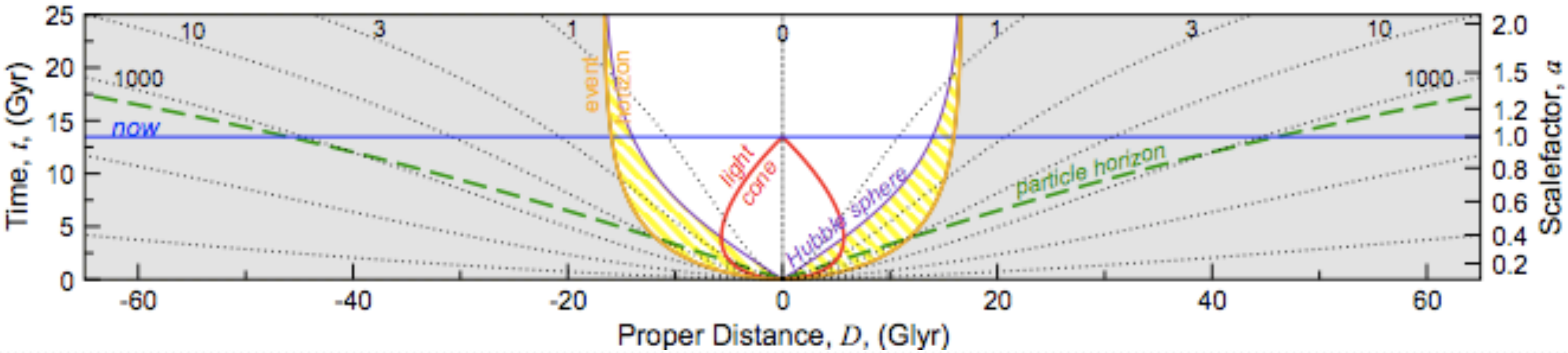
Parámetro densidad de materia

$$\rho_m(t) = \rho_v \sinh^{-2}(t/t_v) \qquad \rho_T = \rho_v \tanh^{-2}(t/t_v)$$



$$\Omega_m(t) = \cosh^{-2}(t/t_v)$$

$$\Omega_m(t_0) = \cosh^{-2}(1,27) = 0,27$$



Time, t, (Gyr)
Scalefactor, a
Proper Distance, D, (Glyr)
now
event horizon
light cone
Hubble sphere
particle horizon
1000
10
3
1
0

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El futuro $t >> t_v$

$$a(t) = A \sinh^{2/3}(t/t_v)$$

$$\sinh x \to \frac{e^x}{2} \quad a(t) \approx B \exp(\sqrt{\Omega_{v,0}} H_0 t)$$

$$H(t) = \frac{2}{3t_v} \coth(t/t_v)$$

$$\coth x \to 1 \quad H(t) \approx \sqrt{\Omega_{v,0}} H_0$$

$$\Omega_m(t) = \cosh^{-2}(t/t_v)$$

$$\cosh^{-2} x \to 0 \quad \Omega_m \approx 0$$

$$\Omega_v(t) = \tanh^2(t/t_v)$$

$$\tanh x \to 1 \quad \Omega_v \approx 1$$

$$q(t) = \frac{1}{2}[1 - 3\tanh^2(t/t_v)]$$

$$\tanh x \to 1 \quad q(t) \approx -1$$

$$\dot{N}_H = 3N_H H(t) q(t)$$

$$\dot{N}_H = -3N_H \sqrt{\Omega_{v,0}} H_0 \Rightarrow N_H(t) = N_1 e^{-\sqrt{\Omega_{v,0}} H_0 t}$$

THE
END!

LAS FUNCIONES HIPERBÓLICAS

- Hyperbolic sine:

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x}$$

- Hyperbolic cosine:

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x}$$

- Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$\operatorname{arsinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\operatorname{arcosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right) ; x \geq 1$$

$$\operatorname{artanh}(x) = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right) ; |x| < 1$$

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \operatorname{arsinh} x = \frac{1}{\sqrt{x^2 + 1}}$$