Putting gravity in control

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Gravity

A dynamical theory of matter and geometry

Control Theory

A way in which an otherwise arbitrary process does what we want.

Free dynamics [Physics]

Natural evolution

Given a Lagrangian L, find the integral curves of E[L] = 0

Controlled dynamics [Engineering]

Desired evolution

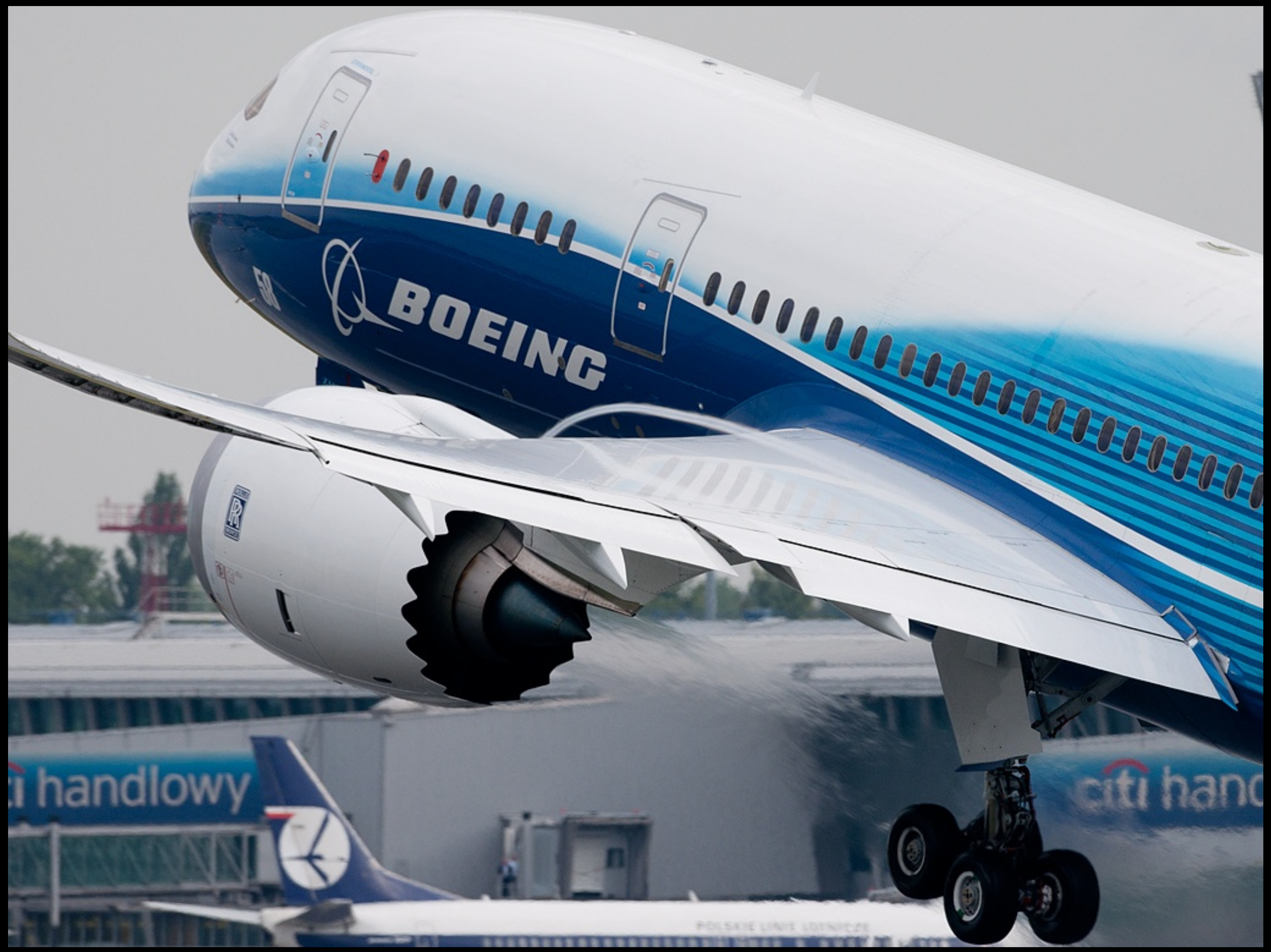
Given a Lagrangian L, find a set of forces u, so that the integral curves of E[L] = u satisfy certain desired property

Stabilization of simple mechanical systems

This is hard!

Stabilization of simple mechanical systems controlling just a few DoF

This is harder!



BOEING
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Geometry Shaping

Simple forced mechanical system

$$\Sigma = \{Q, G, V, F_e\}$$

Configuration space

Riemannian metric (Kinetic energy)

A function on Q (Potential Energy)

External Forces

$$\nabla^{G}_{\dot\gamma(t)}\dot\gamma(t) = -G^{\sharp}\circ \mathrm{d}V\left[\dot\gamma(t)\right] + G^{\sharp}\circ F_e\left[\dot\gamma(t)\right]$$

Acceleration

Potential difference

External forces

Simple mechanical control system

$$\Sigma = \{\underbrace{Q, G, V, F_e,}_{\text{Same as before}} \mathcal{W}\}$$

sub-bundle of the cotangent bundle of Q

$$\nabla^{G}_{\dot{\gamma}(t)}\dot{\gamma}(t) = -G^{\natural}\circ \mathrm{d}V\left[\dot{\gamma}(t)\right] + G^{\natural}\circ F_e\left[\dot{\gamma}(t)\right] + G^{\natural}\circ u\left[\dot{\gamma}(t)\right]$$

Acceleration

Potential difference

External forces

Control input

$$\nabla^{G}_{\dot\gamma(t)}\dot\gamma(t) = -G^{\sharp}\circ \mathrm{d}V\left[\dot\gamma(t)\right] \qquad + G^{\sharp}\circ u\left[\dot\gamma(t)\right]$$

Acceleration

Potential difference

External forces

Control input

Jacobi metric

$$\nabla^{\overset{\star}{G}}_{\dot{\gamma}(t)}\dot{\gamma}(t) =$$ [redacted] $$+\overset{\star}{G}{}^{\natural} \circ u\left[\dot{\gamma}(t)\right]$$

Acceleration

Potential difference

External forces

Control input

Jacobi metric

$$\nabla^{\overset{\star}{G}}_{\dot{\gamma}(t)}\dot{\gamma}(t) =$$ [redacted] [redacted] $$+\,\overset{\star}{G}{}^{\natural} \circ u\left[\dot{\gamma}(t)\right]$$

Acceleration

Potential difference

External forces

Control input

(Underactuated)

Open loop control &
Closed loop control

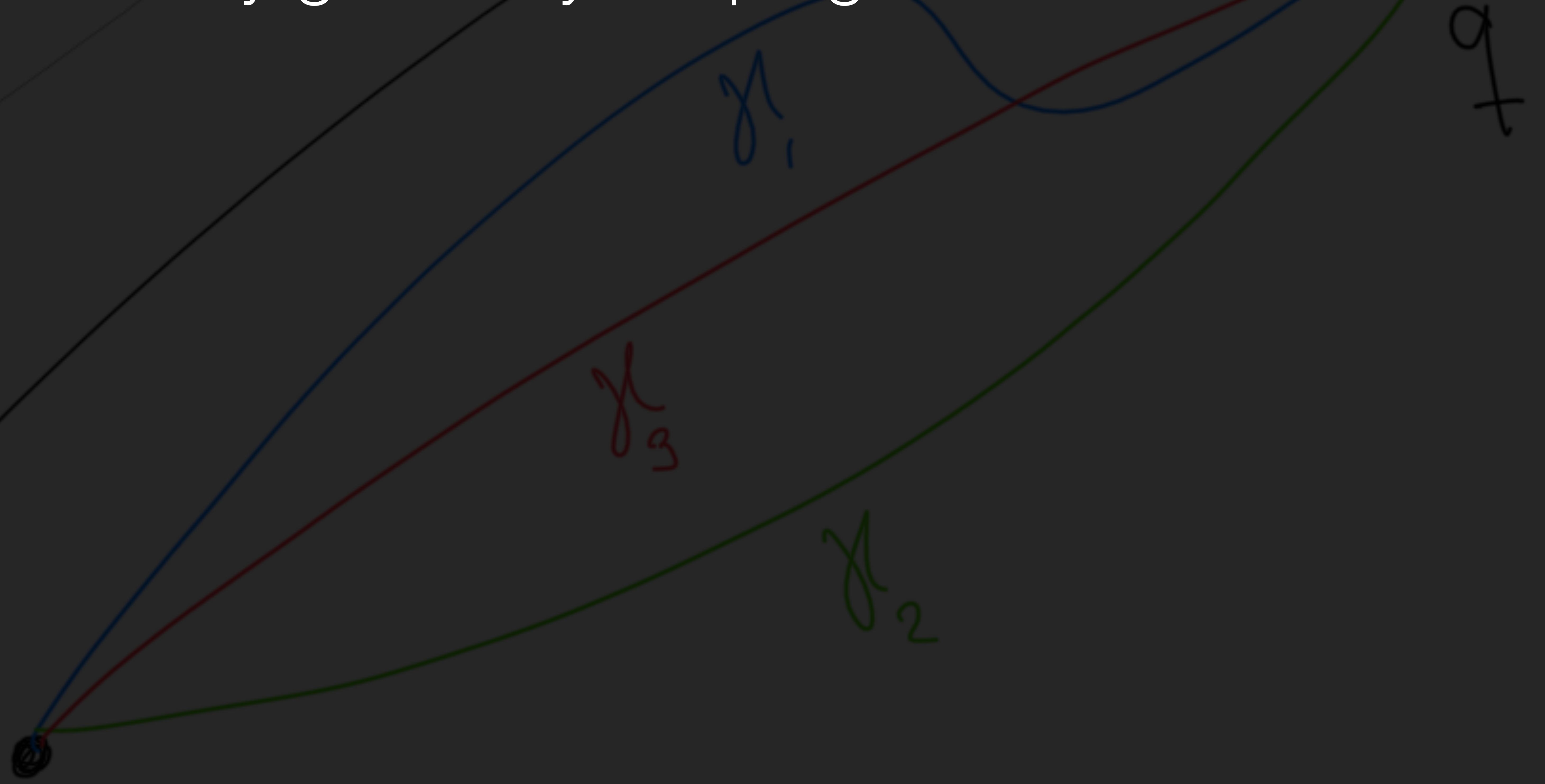
Geometry Shaping

Statement of the problem

Given an open loop simple controlled mechanical system, find (if possible) a control force such that the closed loop system is a forced mechanical system alone

2011-2016

Every linearly controllable simple mechanical system with one degree of under actuation is stabilizable by geometry shaping



There is an infinite set of control laws that do the job.

Which is the best?

Extremize a cost functional

$$J = \int L(G, G', G'', \ldots)$$

Extremize a cost functional

$$J = \int L(G, G', G'', \ldots)$$

This is gravity!

Summary

- The problem of geometrizing dynamics is a contemporary research topic
- Stabilizing simple Lagrangian systems remains largely an open issue
- Geometric control theory is still in its infancy
- Optimal control by energy shaping is morally tantamount to study various gravitational theories.