Colapso y estabilidad de objetos compactos con Ecuación de Estado no Local en Relatividad General

$$P_r(r) = \rho(r) - \frac{2}{r^3}\int_0^r \bar{r}^2 \rho(\bar{r})\, d\bar{r}$$

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Contenido Local

- La ecuación de Estado Nolocal
- De dónde viene y dónde se ha usado
- Algunas Soluciones estáticas
- Criterios de estabilidad
- Fracturas como criterio de estabilidad
- Paréntesis de carga a dos coordenadas
- Colapso de esferas no locales

$$P_r(r) = \rho(r) - \frac{2}{r^3}\int_0^r \bar{r}^2 \rho(\bar{r})\ \mathrm{d}\bar{r}$$

Ecuación de Estado Nolocal

$$P_r(r) = \rho(r) - \frac{2}{3}\left\langle \rho \right\rangle_r$$

$$\left\langle \rho \right\rangle_r = \frac{\int_0^r 4\pi \bar{r}^2 \rho(\bar{r})\ \mathrm{d}\bar{r}}{\frac{4\pi}{3} r^3} = \frac{M(r)}{V(r)}$$

$$P_r(r) = \mathcal{P}(r) + 2\sigma_{\mathcal{P}(r)}$$

$$\mathcal{P}(r) = \frac{1}{3}\rho(r) \qquad \sigma_{\mathcal{P}(r)} = \left(\mathcal{P}(r) - \bar{\mathcal{P}}(r)\right)$$

$$\mathbf{T}_{\mu\nu} = (\rho + P_\perp)\mathbf{u}_\mu \mathbf{u}_\nu - P_\perp \mathbf{g}_{\mu\nu} + (P_r - P_\perp)\mathbf{n}_\mu \mathbf{n}_\nu$$

$$G_{\mu\nu} = T\mu\nu$$

$$\mathrm{d}s^2 = e^{2\nu(r)}\mathrm{d}t^2 - e^{2\lambda(r)}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2$$

$$P_r(r) = \rho(r) - \frac{2}{r^3}\int_0^r \bar{r}^2\rho(\bar{r})\ \mathrm{d}\bar{r}$$

$$g_{\mu\nu}$$

$$\mathrm{d}s^2 = e^{2\beta(r)}\left(\mathrm{d}t^2 - dr^2\right) - r^2\mathrm{d}\Omega^2$$

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NEUTRON STAR MOMENTS OF INERTIA

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ABSTRACT

An approximation for the moment of inertia of a neutron star in terms only of its mass and radius is pre sented, and insight into it is obtained by examining the behavior of the relativistic structural equations. Th approximation is accurate to $\sim$10% for a variety of nuclear equations of state, for all except very low mas stars. It is combined with information about the neutron-star crust to obtain a simple expression (again i terms only of mass and radius) for the fractional moment of inertia of the crust.

Subject headings: equation of state — stars: interiors — stars: neutron — stars: rotation

The equation of state, $P = P(\rho)$, is one of the constraints in the equations solved to obtain the above result. As an alternative stellar model, one can require of equation (7) that the quantity $\Lambda(r)j(r)$ be exactly constant, i.e., that

$$4\pi r^3\left[\rho(r) - \frac{P(r)}{c^2}\right] - 2m(r) = 0\ , \quad (0 < r < R) \qquad (9)$$

in place of the equation of state. (All of the other equations are as before.) An alternative way of writing this equation is

$$P(r) = [\rho(r) - \tfrac{2}{3}\bar{\rho}(r)]c^2\ , \qquad (10)$$

where $\bar{\rho}(r) = m(r)/(4\pi r^3/3)$ may be regarded as some average density over the region interior to r. We thus see that equation (9) is equivalent to a nonlocal equation of state, since the pressure at any point r depends on the density at all points interior to r. A family of homologous stellar shapes results, whose vital

the whole-star quantities M, R, and I. In Figure 1 we show the dependence of I and R on M for one equation of state, FPS (see footnote 1). One of the accompanying curves on that figure represents a successful attempt to guess such a relationship: we find that over a wide range of M, I can be approximated by

$$I \simeq 0.21MR^2\Lambda(R) = 0.21\,\frac{MR^2}{1 - 2GM/Rc^2}\ . \qquad (6)$$

(We recall that for an incompressible fluid in the Newtonian limit $I/MR^2 = \frac{2}{5} = 0.4$. The general relativistic result has been explored by Chandrasekhar & Miller [1974], and its representation in terms of our variables is given below in Fig. 4.) We now try to see under what range of conditions equation (6) may be expected to hold.

In Figure 2 are shown some of the radial functions that are ingredients in a moment of inertia calculation. They are for the FPS equation of state (see footnote 1) and a star mass of $M = 1.445\ M_\odot$, for which $R = 10.8$ km and $\Lambda(R) = 1.65$, a quite relativistic object. The metric-related functions $\Lambda(r)$, $j(r)$, and $\varpi(r)/\Omega$ displayed in Figure 2*a* are seen to be not constant, nor close to one (their Newtonian limit). In view of that fact, it is perhaps surprising that, as is shown in Figure 2*a*, the products $\Lambda(r)j(r)$ and $j(r)\varpi(r)/\Omega$, and thus the ratio $[\varpi(r)/\Omega]/\Lambda(r)$, are remarkably constant in the interior of the star. As to the reason for this, it is straightforward to show that

$$\frac{d}{dr}\Lambda(r)j(r) = \Lambda(r)j(r)\,\frac{G\Lambda(r)}{r^2c^2}\left\{4\pi r^3\left[\rho(r) - \frac{P(r)}{c^2}\right] - 2m(r)\right\}. \qquad (7)$$

The behavior of the various quantities on the right of this equation are plotted in Figure 2*b*. If they are evaluated at $r \to 0$, the equation becomes

$$\frac{d}{dr}\Lambda(r)j(r) = 2\Lambda(0)j(0)\,\frac{G\Lambda(0)}{c^2}\,4\pi r\rho(0)\left[\frac{1}{3} - \frac{P(0)}{\rho(0)c^2}\right], \quad (r \to 0)\ . \qquad (8)$$

For the case illustrated, the second ratio in the square bracket has the value -0.184. For a star mass of $M = 1.70\ M_\odot$, it has

A non commutative model for a mini black hole

Ivan Arraut Guerrero, Davide Batic, Marek Nowakowski

(Submitted on 19 Feb 2009 (v1), last revised 6 Apr 2009 (this version, v3))

We analyze the static and spherically symmetric perfect fluid solutions of Einstein field equations inspired by the non commutative geometry. In the framework of the non commutative geometry this solution is interpreted as a mini black hole which has the Schwarzschild geometry outside the event horizon, but whose standard central singularity is replaced by a self-gravitating droplet. The energy-momentum tensor of the droplet is of the anisotropic fluid obeying a nonlocal equation of state. The radius of the droplet is finite and the pressure, which gives rise to the hydrostatic equilibrium, is positive definite in the interior.

Comments:	10 pages, 2 figures, bibliography enlarged, reference in the conclusion fixed, some typos corrected
Subjects:	**General Relativity and Quantum Cosmology (gr-qc)**
Journal reference:	Class.Quant.Grav.26:245006,2009
DOI:	10.1088/0264-9381/26/24/245006
Cite as:	**arXiv:0902.3481v3 [gr-qc]**

Submission history

From: Davide Batic [view email]

Radial pulsations and stability of anisotropic stars with quasi-local equation of state

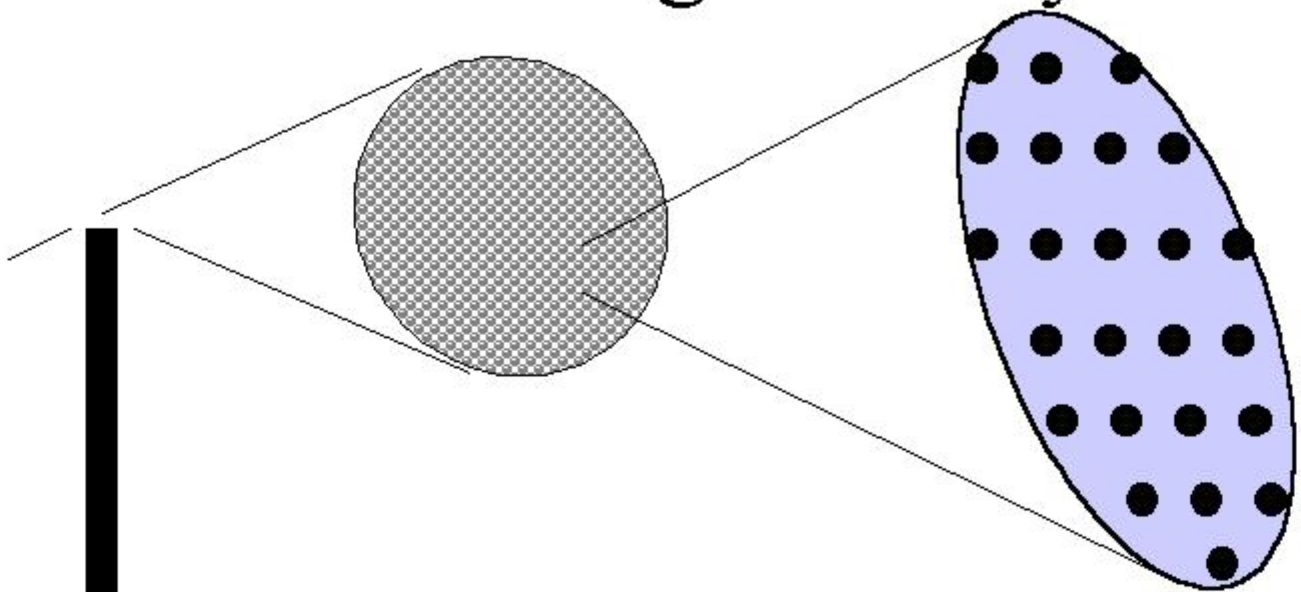
Dubravko Horvat, Sasa Ilijic, Anja Marunovic

(Submitted on 5 Oct 2010)

Quasi-local variables, i.e. quantities whose values can be derived from physics accessible within an arbitrarily small neighborhood of a spacetime point, are used to construct the equation of state for the anisotropic fluid in spherical symmetry. One parameter families of equilibrium solutions are obtained making it possible to assess stability properties by means of the standard M(R) method. Normal modes of radial pulsation are computed as well and are found to confirm the onset of instability as predicted by the M(R) method. As an example, a stable configuration with outwardly increasing energy density in the core is obtained with a simple quasi-local extension of the polytropic equation of state. It is also found that the loss of stability occurs at higher surface compactness when the anisotropy of pressures is present.

Comments:	15 pages, 3 figures
Subjects:	**General Relativity and Quantum Cosmology (gr-qc)**
Cite as:	**arXiv:1010.0878v1 [gr-qc]**

No-local Damage Theory



$$\left\langle \begin{array}{l} F = \boldsymbol{Kx} \\ F^{e} = \dfrac{F}{1-\Omega} \end{array} \right\rangle \Rightarrow F = \left(1-\Omega\right)Kx$$

$$\Omega(x) = \frac{1}{\int_V \alpha\left(s-x\right)\mathrm{d}V(s)} \int_V \alpha\left(s-x\right)\omega(s)\,\mathrm{d}V(s)$$

Las Ecuaciones de Einstein

$$8\pi\rho = \frac{2m'}{r^2}$$

$$8\pi P_r = \frac{2m'}{r^2} - \frac{4m}{r^3}$$

$$8\pi P_\perp = \frac{m''}{r} + \frac{2\left(m'r-m\right)}{r^3}\left[\frac{m'r-m}{r-2m} - 1\right]$$

$$\rho = \rho(r) \Rightarrow m(r) = 4\pi \int_0^r \rho\, \bar{r}^2\, \mathrm{d}\bar{r}$$

$$P_r = 0 = \frac{2m'}{r^2} - \frac{4(m-\mathcal{C})}{r^3} \Rightarrow m(r)$$

$$P_\perp = 0 = \frac{m''}{r} + \frac{2\left(m'r-m\right)}{r^3}\left[\frac{m'r-m}{r-2m} - 1\right] \Rightarrow m(r)$$

Algunos ejemplos de perfiles de densidad

$$\rho_S = \frac{1}{8\,\pi r^2}\,\frac{\left(e^{2Kr}-1\right)\left(e^{4Kr}+8Kre^{2Kr}-1\right)}{\left(e^{2Kr}+1\right)^3} \quad \Leftrightarrow \quad m_S = \frac{r}{2}\left(\frac{e^{2Kr}-1}{e^{2Kr}+1}\right)^2$$

Stewart, BW (1982) *J Phys. A. Math Gen.,* **15,** 2419.

$$\rho_{GM} = \frac{\sigma}{8\pi}\left[1 - K\,\frac{r^2}{a^2}\right] \quad \Leftrightarrow \quad m_{GM} = \frac{\sigma\, r^3}{6}\,\left[1\ -\ \frac{3\,K}{5}\,\frac{r^2}{a^2}\right]$$

Gokhroo, MK and Mehra, AL (1994), *Gen. Rel. Grav.,* **26**, 75.

$$\rho_W = -\frac{C}{8\,\pi}\,\frac{K\left(3+5\,r^2\right)}{(1+3r^2)^{\frac{5}{3}}} \quad \Leftrightarrow \quad m_S = -\frac{1}{2C^{\frac{1}{2}}}\,\frac{Kr^3}{(1+3r^2)^{\frac{2}{3}}}\ ,$$

Wyman, M., (1949), *Phys. Rev.* **75**, 1930

Perfil Densidad	μ	$M\ (M_\odot)$	z_a	$\rho_a \times 10^{14}\,(gr/cm^3)$	$\rho_c\ \times 10^{15}\ (gr/cm^3)$
Stewart	0.32	2.15	0.6	6.80	1.91
Gokhroo-Mehra	0.40	2.80	1.2	8.84	1.99
Wyman	0.38	2.54	1.0	8.04	3.04

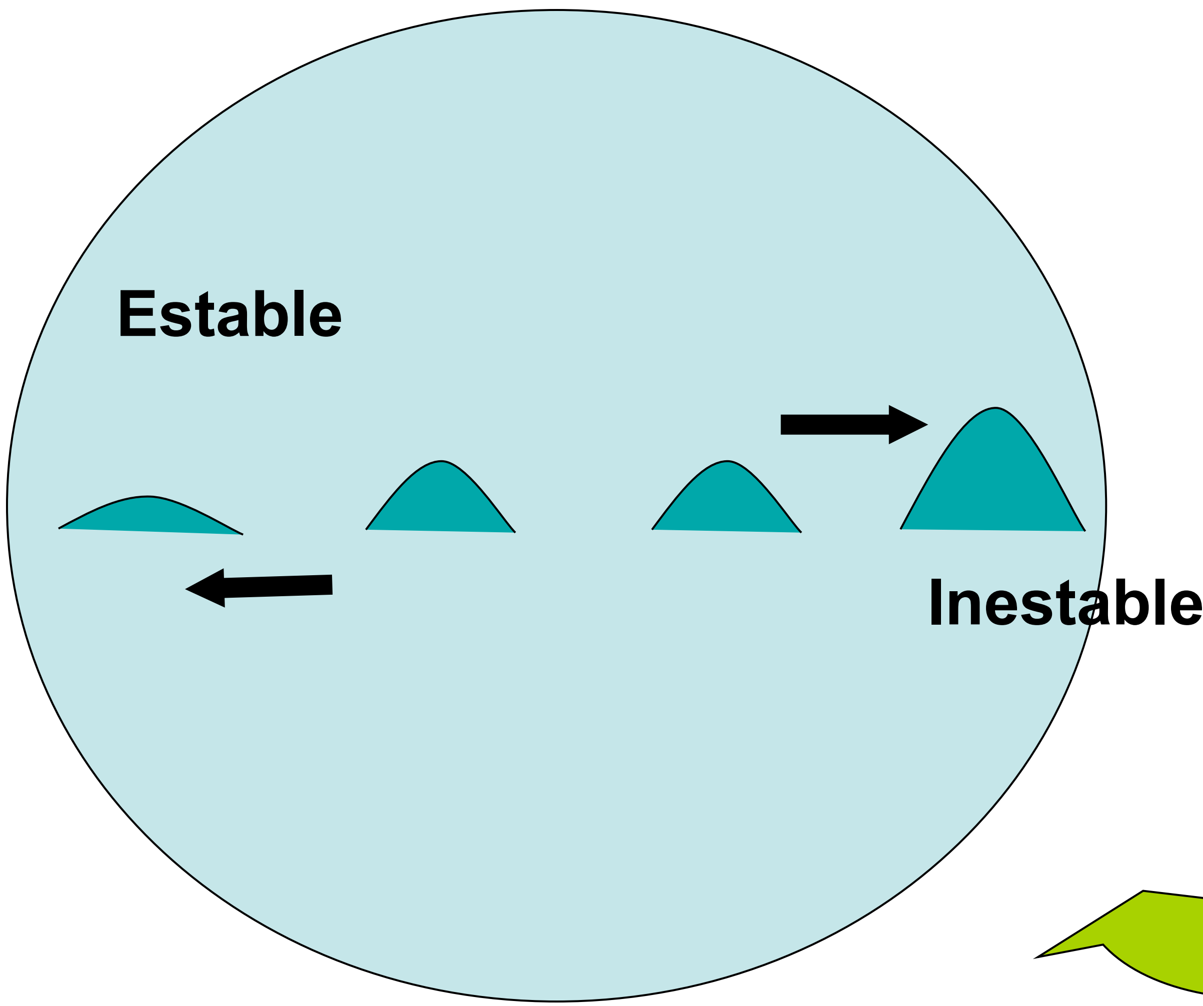
$P_r = 0$ y $P_\perp \neq 0$ $\qquad \frac{2m'}{r^2} = \frac{4m}{r^3} \Rightarrow m\,(r) = Cr^2.$

$P_\perp = 0$ y $P_r \neq 0$ $\qquad \frac{m''}{r} + \frac{2(m'r-m)}{r^3}\left[\frac{m'r-m}{r-2m} - 1\right] = 0$

$m = \frac{r}{2}\left[1 - e^{-2(C_1 r + C_2)}\right]$ $\qquad m_{TW}(r) = Cr^2$

$$m_{TW}\,(r) = 4\pi\int_0^r r^2 e^{(\nu+\lambda)/2}\,(\rho - P_r - 2P_\perp)\,\mathrm{d}r \equiv e^{(\nu+\lambda)/2}\left(m + 4\pi P_r r^3\right)$$

Perturbaciones Dinámicas

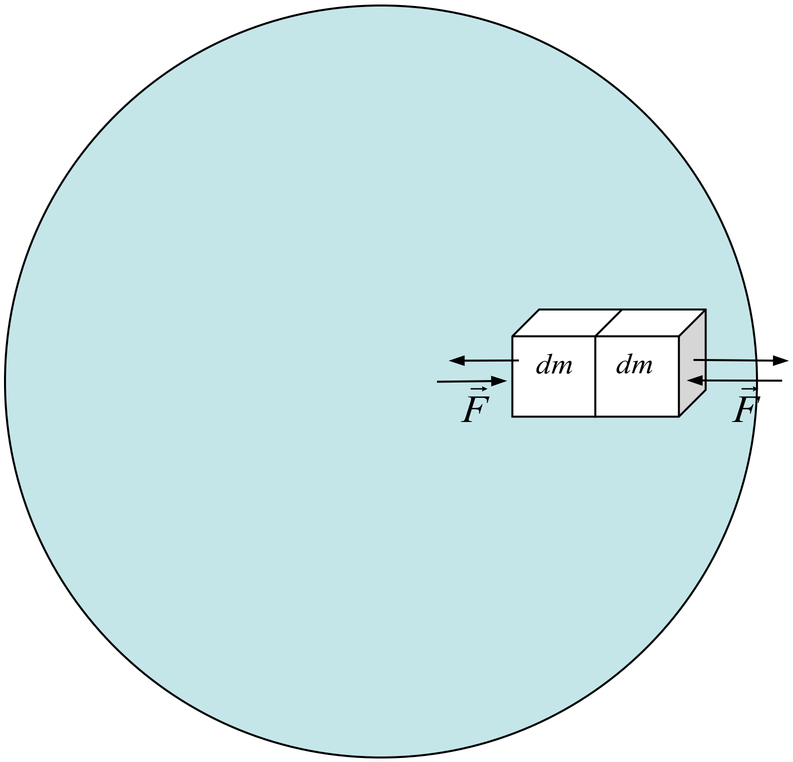


$$\rho(r,t) = \rho_0 + \delta\rho(r,t)$$
$$P_r(r,t) = P_{r0} + \delta P_r(r,t)$$
$$P_\perp(r,t) = P_{\perp 0} + \delta P_\perp(r,t)$$
$$\nu(r,t) = \nu_0 + \delta\nu(r,t)$$
$$\lambda(r,t) = \lambda_0 + \delta\lambda(r,t)$$

Ecuaciones de Einstein

Chandrasekhar C *Astrophys. J.* (1964) **140** 417-433
Dev D y Gleiser G *Gen. Rel. Grav.* 2003 **35**, 1435-1457

Fracturas



$$\rho(r,t) = \rho_0 + \delta\rho(r)$$

$$m_r(r,t) = m_{r0} + \delta m_r(r)$$

$$P_r(r,t) = P_{r0} + \delta P_r(r)$$

$$\Delta(r,t) = \Delta_0 + \delta\Delta(r)$$

$$\mathcal{R} = \frac{\mathrm{d}Pr}{\mathrm{d}r} + (\rho + P_r)\left(\frac{m + 4\pi r^3 P_r}{r\,(r-2m)}\right) - \frac{2}{r}\,(P_\perp - P_r)$$

$$\mathcal{R}(\rho_0{+}\delta\rho, m_0{+}\delta m, P_{r0}{+}\delta P_r \Delta_0{+}\delta\Delta) \approx \underbrace{\mathcal{R}_0(\rho_0, m_0, P_{r0}, \Delta_0)}_{=0} + \tilde{\mathcal{R}}(\rho_0, m_0, P_{r0}, \Delta_0, \delta\rho, \delta m, \delta P_r, \delta\Delta)$$

Herrera *Phys. Lett A,* **165,** 296 (1992); Di Prisco et al *Phys. Lett A,* **195,** 23 (1994)
Di Prisco, Herrera and Varela *GRG* **29**, 10 (1997)

Fractura y Fuerzas de Marea

L. Herrera (1992)*Cracking of self-gravitating compact objects.* Physics Letts A,**165**, 206.

$$\mathrm{d}s^2 = e^{\lambda(r)}\mathrm{d}t^2 - e^{\nu(r)}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin\theta\mathrm{d}\phi^2)$$

$$\mathcal{R} = \frac{\mathrm{d}Pr}{\mathrm{d}r} + (\rho + P_r)\left(\frac{m + 4\pi r^3 P_r}{r\,(r - 2m)}\right) - \frac{2}{r}\,(P_\perp - P_r)$$

$$a^\alpha = \left[-R^\alpha_{\beta\gamma\mu}u^\beta u^\mu + h^\alpha_\beta\left(\frac{\mathrm{d}u^\beta}{\mathrm{d}s}\right)_{;\gamma} - \frac{\mathrm{d}u^\alpha}{\mathrm{d}s}\frac{\mathrm{d}u_\gamma}{\mathrm{d}s}\right] h^\gamma_\nu\,\delta x^\nu$$

$$\mathcal{R} = -\frac{e^\lambda(\rho + P_r)}{e^{\nu/2}r^2}\int_0^a \mathrm{d}\tilde{r}\; e^{\nu/2}\tilde{r}^2\frac{\mathrm{d}\Theta}{\mathrm{d}s}$$

Fractura y Fuerzas de Marea

$$\mathcal{R}(\rho_0+\delta\rho, m_0+\delta m, P_{r0}+\delta P_r \Delta_0+\delta\Delta) \approx \underbrace{\mathcal{R}_0(\rho_0, m_0, P_{r0}, \Delta_0)}_{=0} + \tilde{\mathcal{R}}(\rho_0, m_0, P_{r0}, \Delta_0, \delta\rho, \delta m, \delta P_r, \delta\Delta)$$

$$\rho + \delta\rho \Rightarrow \begin{cases} P_r(\rho+\delta\rho, r) \approx P_r(\rho, r) + \delta P_r \approx P_r(\rho, r) + \frac{\partial P_r}{\partial \rho}\delta\rho \,, \\ \\ m(\rho+\delta\rho, r) = 4\pi \int_0^r (\rho+\delta\rho)\bar{r}^2 d\bar{r} \approx m(\rho, r) + \frac{4\pi}{3} r^3 \delta\rho \,. \end{cases}$$

$$\tilde{\mathcal{R}} = \delta\rho \left[\left(2\frac{\partial \mathcal{R}}{\partial \rho} + \frac{4\pi}{3} r^3 \frac{\partial \mathcal{R}}{\partial m} \right) - \frac{2}{r}\frac{\delta\Delta}{\delta\rho} \right]$$

$$\frac{\partial \mathcal{R}}{\partial \rho} = \frac{m + 4\pi\, P_r r^3}{r(r-2m)} \geq 0 \quad \text{y} \quad \frac{\partial \mathcal{R}}{\partial m} = \frac{(\rho + P_r)\left(1 + 8\pi\, P_r r^2\right)}{(r - 2\,m)^2} \geq 0$$

entonces

$$\rho(r) \to P_r = P_r(\rho(r)) \to P_\perp = P_r + \frac{r}{2}\frac{\mathrm{d}P_r}{\mathrm{d}r} + \frac{(\rho + P_r)}{2}\left(\frac{m + 4\pi r^3 P_r}{(r-2m)}\right)$$

Programa de Perturbaciones y Fracturas

$$\rho(r) \rightarrow \begin{cases} \rho(r) - \frac{2}{r^3}\int_0^r \bar{r}^2 \rho(\bar{r})\, \mathrm{d}\bar{r} + \frac{\mathcal{C}}{2\pi r^3} = P_r(r) \\ K\rho^{\Gamma}(r) = P_r(r) \end{cases} \quad \text{para } R = 0 \Rightarrow \begin{cases} \Delta_{NL} \\ \Delta_{Politropa} \end{cases}$$

$$\rho_S = \frac{1}{8\,\pi r^2} \frac{\left(e^{2Kr}-1\right)\left(e^{4Kr}+8Kre^{2Kr}-1\right)}{\left(e^{2Kr}+1\right)^3} \quad \Leftrightarrow \quad m_S = \frac{r}{2}\left(\frac{e^{2Kr}-1}{e^{2Kr}+1}\right)^2$$

Stewart, BW (1982) *J Phys. A. Math Gen.,* **15,** 2419.

$$\rho_{GM} = \frac{\sigma}{8\pi}\left[1 - K\frac{r^2}{a^2}\right] \quad \Leftrightarrow \quad m_{GM} = \frac{\sigma\, r^3}{6}\left[1 - \frac{3\,K}{5}\frac{r^2}{a^2}\right]$$

Gokhroo, MK and Mehra, AL (1994), *Gen. Rel. Grav.,* **26**, 75.

$$\rho_W = -\frac{C}{8\,\pi}\frac{K\left(3+5\,r^2\right)}{(1+3r^2)^{\frac{5}{3}}} \quad \Leftrightarrow \quad m_S = -\frac{1}{2C^{\frac{1}{2}}}\frac{Kr^3}{(1+3r^2)^{\frac{2}{3}}}\ ,$$

Wyman, M., (1949), *Phys. Rev.* **75**, 1930

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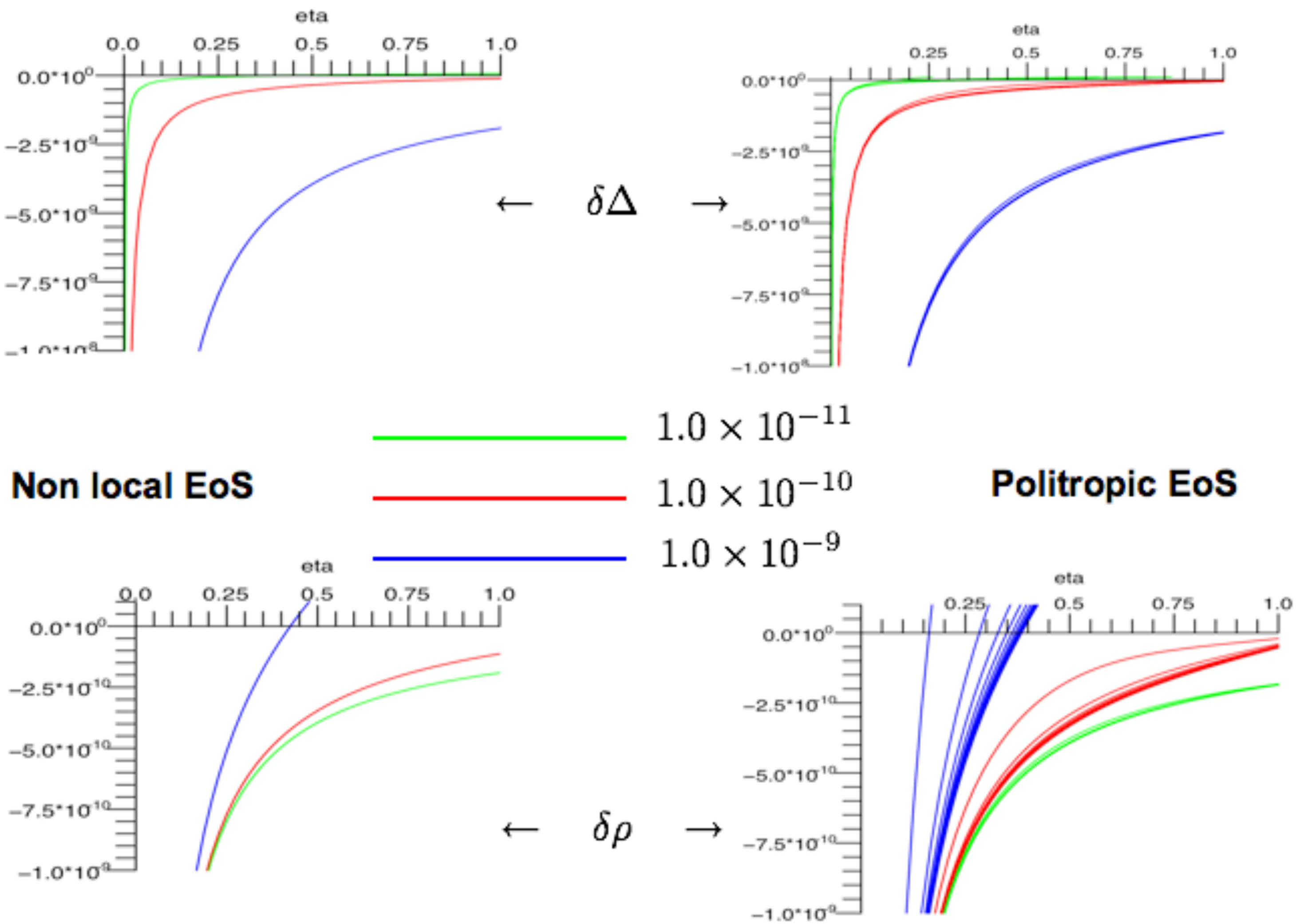
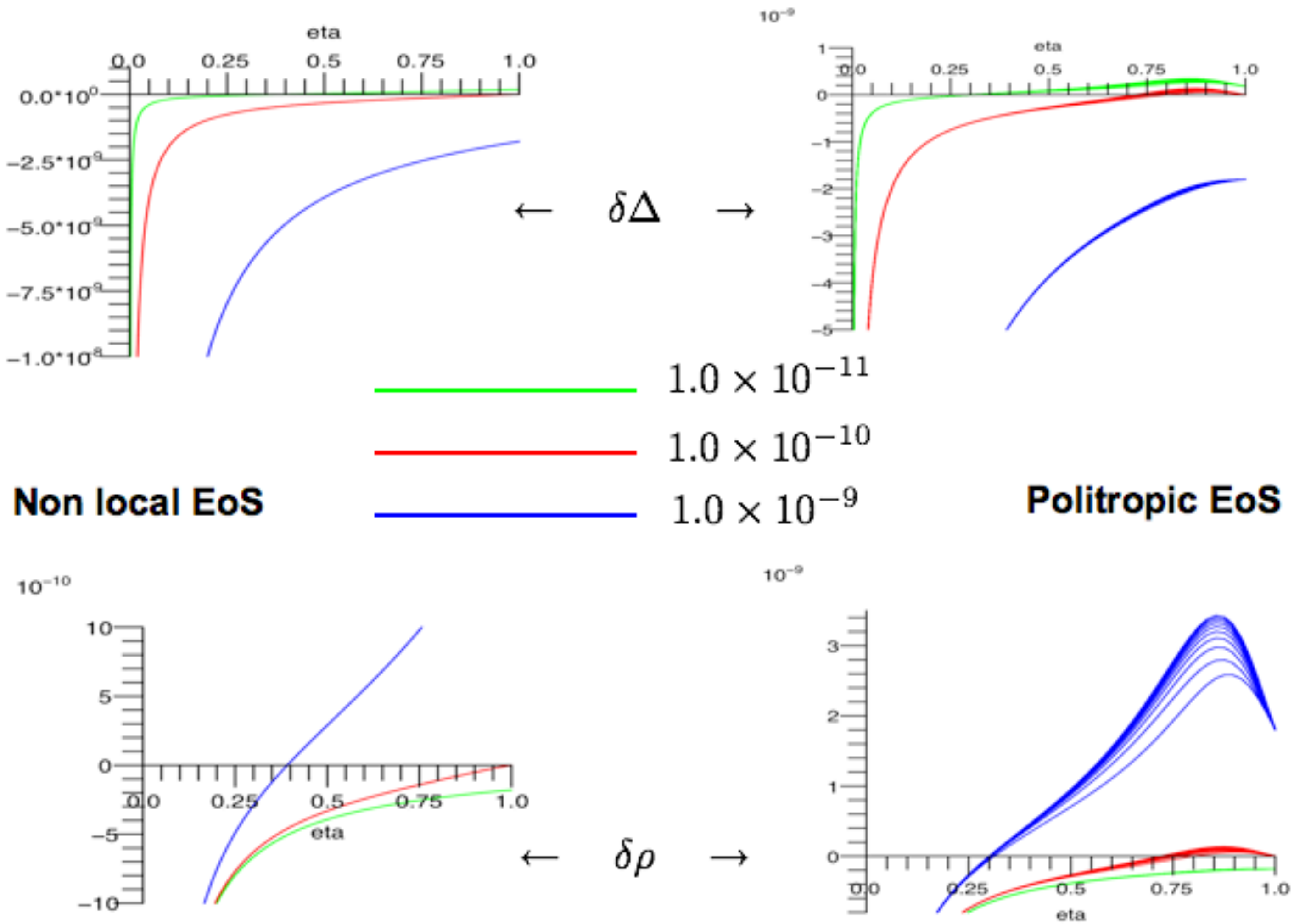
Stewart	0.32	2.15	0.6	6.80	1.91
Gokhroo-Mehra	0.40	2.80	1.2	8.84	1.99
Wyman	0.38	2.54	1.0	8.04	3.04

Ejemplos de Perturbaciones y Fracturas



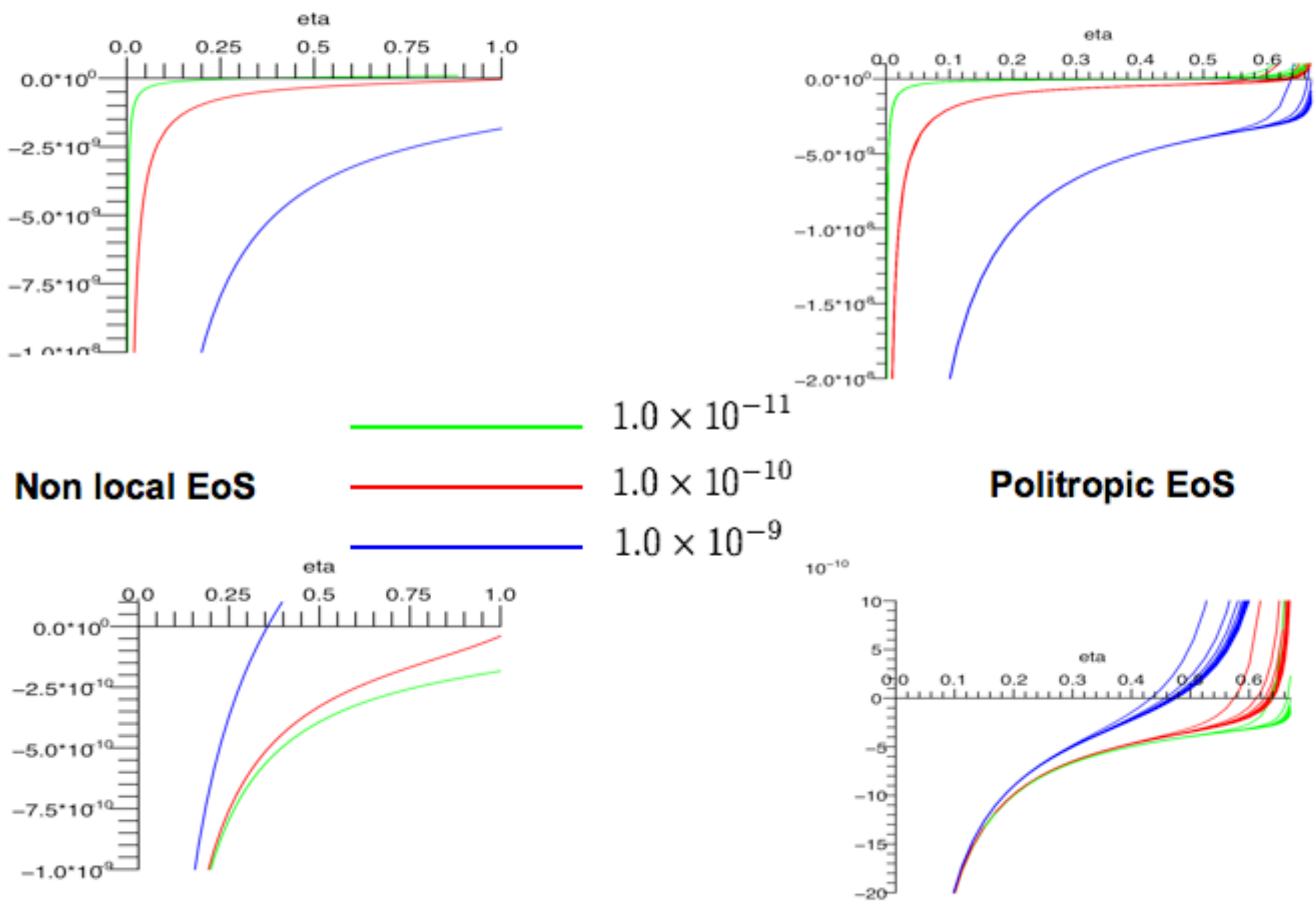
Cracking for Stewart Density Profile

Ejemplos de Perturbaciones y Fracturas



Cracking for Gokhroo-Mehra Density Profile

Ejemplos de Perturbaciones y Fracturas



Cracking for Wyman Density Profile

Abreu Hernández y Núñez. (2007) *J. Physics: Conf Series*, **66**, 2007.

Velocidades del Sonido y perturbaciones

$$\tilde{\mathcal{R}} = \delta\rho \left[\left(2 \frac{\partial \mathcal{R}}{\partial \rho} + \frac{4\pi}{3} r^3 \frac{\partial \mathcal{R}}{\partial m} \right) - \frac{2}{r} \frac{\delta \Delta}{\delta \rho} \right]$$

$$\frac{\delta \Delta}{\delta \rho} \sim \frac{\delta \left(P_\perp - P_r \right)}{\delta \rho} \sim \frac{\delta P_\perp}{\delta \rho} - \frac{\delta P_r}{\delta \rho} \sim v_{s\perp}^2 - v_{sr}^2$$

$$-1 \leq v_{s\perp}^2 - v_{sr}^2 \leq 1 \Rightarrow \begin{cases} -1 \leq v_{s\perp}^2 - v_{sr}^2 \leq 0 & \text{Potencialmente estable ,} \\ 0 < v_{s\perp}^2 - v_{sr}^2 \leq 1 & \text{Potencialmente inestable .} \end{cases}$$

Velocidades del Sonido y perfiles

$$\rho = \frac{K}{r^2}, \quad \Rightarrow P_r = \frac{3}{8\pi r^2}\left(\frac{1-\sqrt{\frac{r}{a}}}{7-3\sqrt{\frac{r}{a}}}\right),$$

Tolman R C. (1939) *Physical Review*, **55**, 364,

$$\rho = \frac{1}{8\pi r^2}\left[1 - \frac{\sin(2Kr)}{Kr} + \frac{\sin^2(Kr)}{K^2 r^2}\right]$$

$$\rho_S = \frac{1}{8\,\pi r^2}\frac{\left(e^{2Kr}-1\right)\left(e^{4Kr}+8Kre^{2Kr}-1\right)}{(e^{2Kr}+1)^3} \quad \Leftrightarrow \quad m_S = \frac{r}{2}\left(\frac{e^{2Kr}-1}{e^{2Kr}+1}\right)^2$$

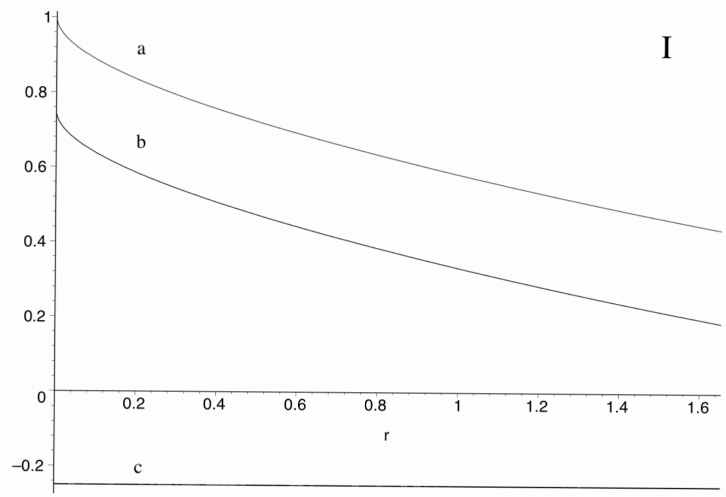
Stewart, BW (1982) *J Phys. A. Math Gen.,* **15,** 2419.

$$\rho_{GM} = \frac{\sigma}{8\pi}\left[1 - K\frac{r^2}{a^2}\right] \quad \Leftrightarrow \quad m_{GM} = \frac{\sigma\, r^3}{6}\left[1\; - \frac{3\,K}{5}\frac{r^2}{a^2}\right]$$

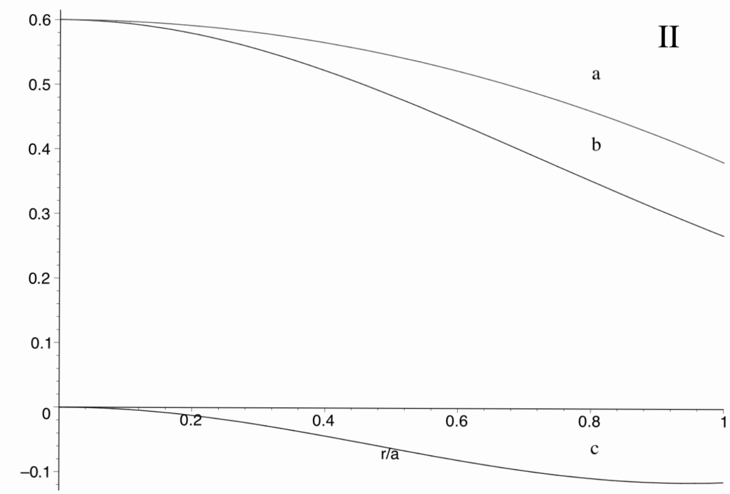
Gokhroo, MK and Mehra, AL (1994), *Gen. Rel. Grav.,* **26**, 75.

Perfil Densidad	M/a	$M(M_\odot)$	z_a	$\rho_a \times 10^{14}\,(gr/cm^3)$	$\rho_c \times 10^{15}\,(gr/cm^3)$
Tolman VI	0.21	1.42	0.31	2.30	NA
NL Stewart 1	0.32	2.15	0.65	6.80	1.91
NL Stewart 2	0.39	2.68	1.19	8.49	2.14
Gokhroo & Mehra	0.26	1.76	0.44	0.00	2.09

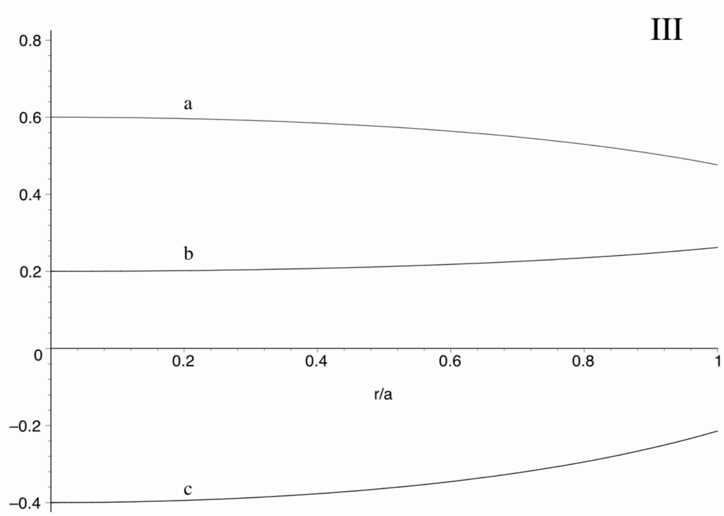
Velocidades del Sonido y modelos



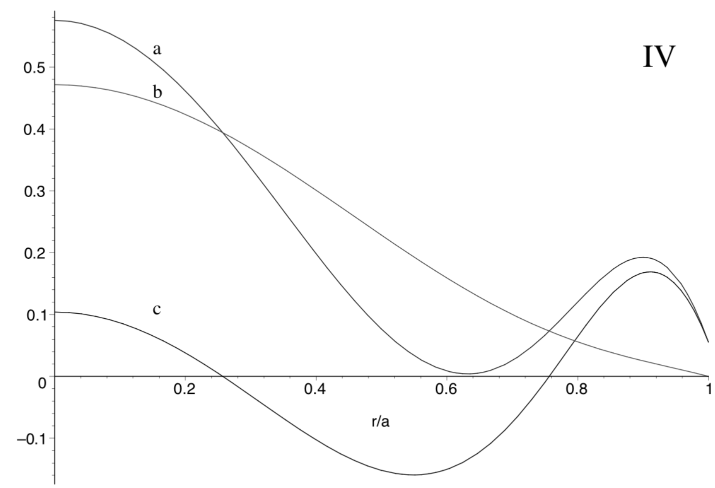
Velocidades sonido TolmanVI



Velocidades sonido Steward 1

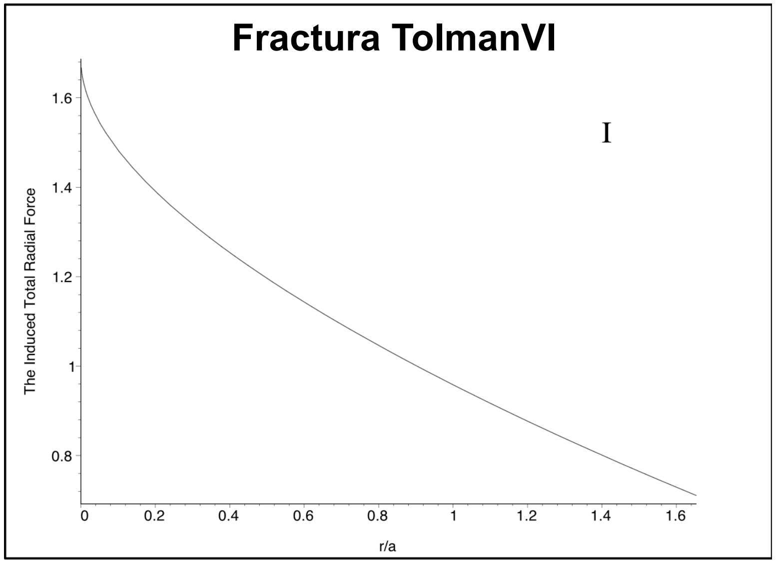


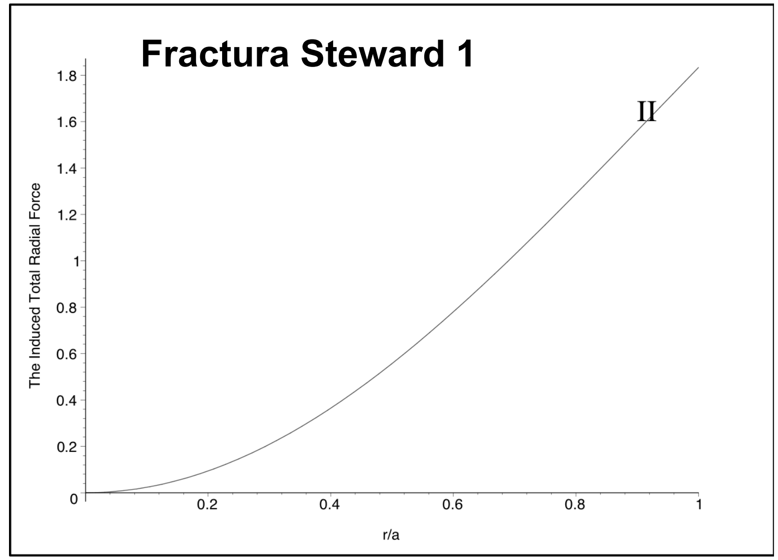
Velocidades sonido Steward 2

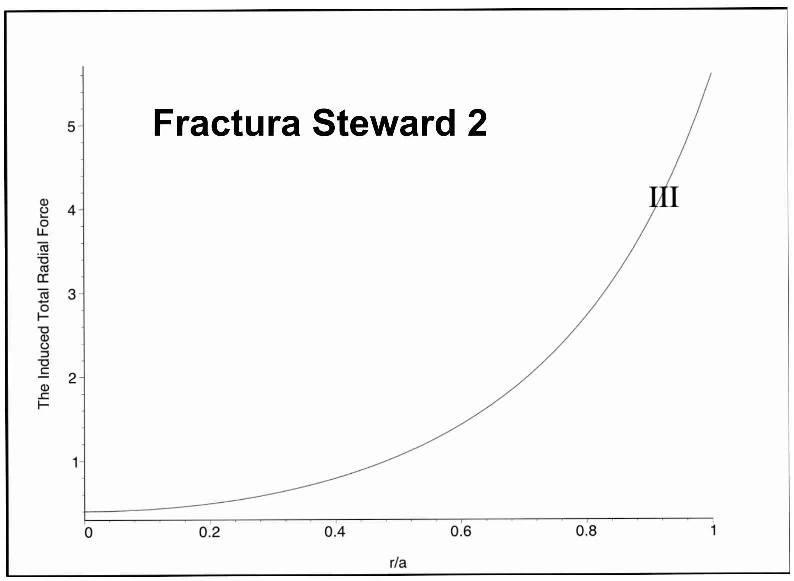


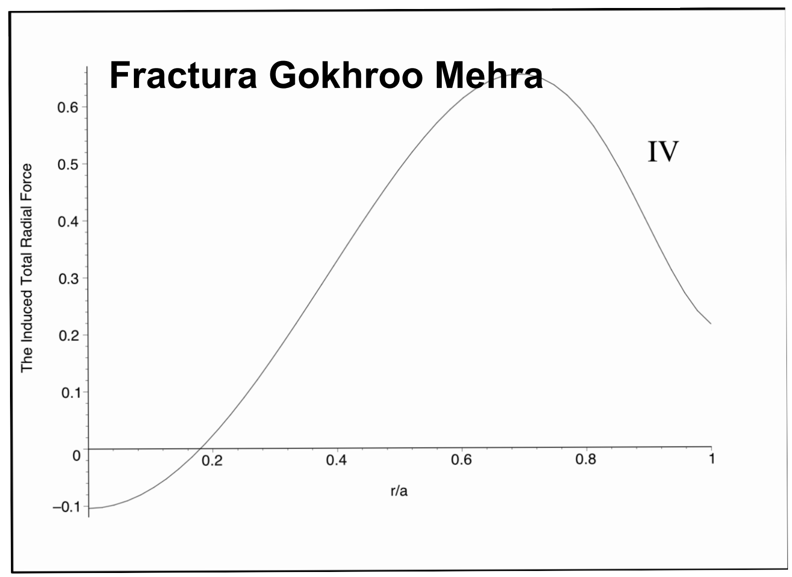
Velocidades sonido Gokhroo Mehra

Velocidades del Sonido y fracturas









Abreu Hernández y Núñez. (2007) *Class. Quant. Grav.*, **24**:4631

Carga y Perturbaciones de Carga

$$ds^2 = -e^{2\lambda(r)}dt^2 + e^{2\nu(r)}dr^2 + r^2(d\theta^2 + \sin\theta d\phi^2)$$

$$T^{\nu\mu} = (\rho + P_t)\, u^\mu u^\nu - P_t g^{\mu\nu} + (P_r - P_t)\, s^\mu s^\nu + \frac{1}{4}\pi \left[F^{\mu\alpha} F^\nu_\alpha - \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}\right]$$

$$\frac{\partial\left(r^2 e^\alpha F^{01}\right)}{\partial r} = 4\pi j^0 r^2 e^\alpha \quad \Rightarrow F^{01} = e^{-\alpha}\frac{Q(r)}{r^2} \quad \text{con} \;\; Q(r) = \int_0^r d\bar{r}\; e^\alpha 4\pi \bar{r}^2 j^0.$$

$$\frac{\partial\left(r^2 e^\alpha F^{01}\right)}{\partial t} = -4\pi j^1 r^2 e^\alpha = 0 \quad \text{donde } \alpha = (\lambda + \nu)\ .$$

$$\frac{d P_r}{dr} = \frac{Q}{4\pi r^4}\frac{d Q}{dr} - \frac{(\rho + P_r)\left(4\pi r P_r - \frac{m}{r^2} + \frac{Q^2}{r^3}\right)}{\left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)} + \frac{2\Delta}{r}$$

Carga y Perturbaciones de Carga

$$\mathcal{R}\left(\rho, m, P_r, \Delta, Q, \frac{\mathrm{d}\,Q}{\mathrm{d}r}\right) = \frac{\mathrm{d}\,P_r}{\mathrm{d}\,r} - \frac{Q}{4\pi r^4}\frac{\mathrm{d}\,Q}{\mathrm{d}r} + \frac{(\rho+P_r)\left(4\pi r P_r - \frac{m}{r^2} + \frac{Q^2}{r^3}\right)}{\left(1-\frac{2m}{r}+\frac{Q^2}{r^2}\right)} - \frac{2\Delta}{r}$$

$$\mathcal{R} \approx \underbrace{\mathcal{R}_0\,(\rho, \rho m, P_r, \Delta, \xi, \chi)}_{\mathcal{R}_0=0} + \underbrace{\delta\rho\frac{\partial\mathcal{R}}{\partial\rho} + \delta P_r\frac{\partial\mathcal{R}}{\partial P_r} + \delta m\frac{\partial\mathcal{R}}{\partial m} + \delta\xi\frac{\partial\mathcal{R}}{\partial\xi} + \delta\chi\frac{\partial\mathcal{R}}{\partial\chi} + \delta\Delta\frac{\partial\mathcal{R}}{\partial\Delta}}_{\delta\mathcal{R}}$$

$$\text{con } \chi = Q^2 \text{ y } \xi = \frac{1}{2}\frac{\mathrm{d}\chi}{\mathrm{d}r}$$

$$\delta\mathcal{R} = \delta\Delta\frac{\partial\mathcal{R}}{\partial\Delta} + \delta\chi\left(\frac{1}{2r}\frac{\partial\mathcal{R}}{\partial\xi} + \frac{\partial\mathcal{R}}{\partial\chi}\right) + \delta\rho\left(\frac{\partial P_r}{\partial\rho}\frac{\partial\mathcal{R}}{\partial P_r} + \frac{\partial\mathcal{R}}{\partial\rho} + \frac{4}{3}\pi r^3\frac{\partial\mathcal{R}}{\partial m}\right)$$

$$\delta\mathcal{R} = -\frac{2\delta\Delta}{r} - \left(\frac{1}{8\pi r^5} + \frac{(\rho+P_r)\left(4\pi r^2 P_r + 1 - \frac{m}{r} + \frac{\chi}{r^2}\right)}{\left(1-\frac{2m}{r}+\frac{\chi}{r^2}\right)^2 r^3}\right)\delta\chi$$

$$+ \left(\frac{4\pi r}{3}\frac{(\rho+P_r)(1-\frac{\chi}{r^2}+8\pi r^2 P_r)}{\left(1-\frac{2m}{r}+\frac{\chi}{r^2}\right)^2} + \frac{\left(8\pi r^2 P_r + \frac{2m}{r} - \frac{2\chi}{r^2}\right)}{r\left(1-\frac{2m}{r}+\frac{\chi}{r^2}\right)}\right)\delta\rho$$

Carga y Perturbaciones de Carga

para perturbaciones en objetos neutros, isótropos $\Delta = 0$, $\xi_0 = 0$ y $\chi_0 = 0$,

$$\delta\mathcal{R} = -\frac{2\delta\Delta}{r} - \left(\frac{1}{8\pi r^5} + \frac{(\rho_0 + P_{r0})\left(4\pi r^2 P_{r0} + 1 - \frac{m_0}{r}\right)}{\left(1 - \frac{2m_0}{r}\right)^2 r^3}\right)\delta\chi$$

$$+ \left(\frac{4\pi r}{3}\frac{(\rho_0 + P_{r0})(1 + 8\pi r^2 P_{r0})}{\left(1 - \frac{2m_0}{r}\right)^2} + \frac{\left(8\pi r^2 P_{r0} + \frac{2m_0}{r}\right)}{r\left(1 - \frac{2m_0}{r}\right)}\right)\delta\rho$$

Otra vez

$$\left(\frac{1}{8\pi r^5} + \frac{(\rho_0 + P_{r0})\left(4\pi r^2 P_{r0} + 1 - \frac{m_0}{r}\right)}{\left(1 - \frac{2m_0}{r}\right)^2 r^3}\right) \geq 0$$

y

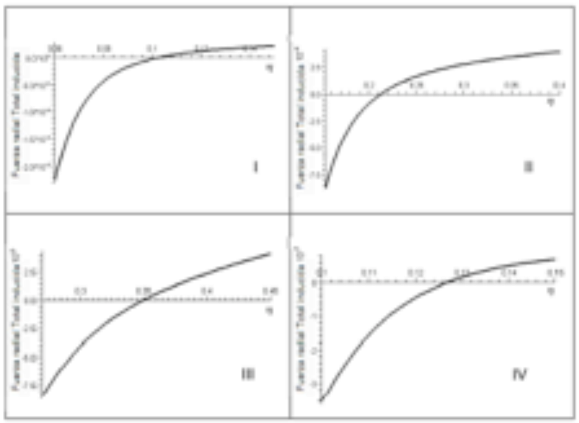
$$\left(\frac{4\pi r}{3}\frac{(\rho_0 + P_{r0})(1 + 8\pi r^2 P_{r0})}{\left(1 - \frac{2m_0}{r}\right)^2} + \frac{\left(8\pi r^2 P_r + \frac{2m_0}{r}\right)}{r\left(1 - \frac{2m_0}{r}\right)}\right) \geq 0$$

Signos de Perturbaciones de carga, anisotropía y densidad

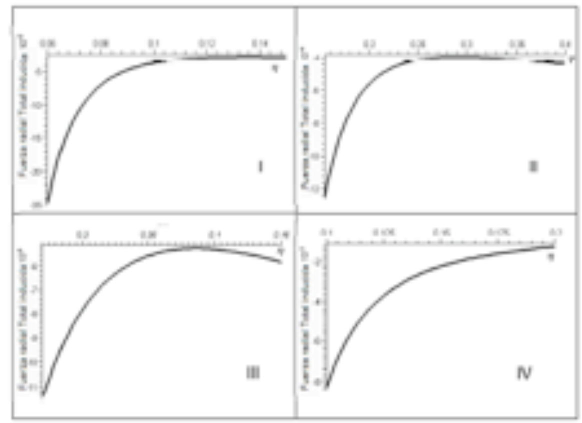
Caso	Signos		
	$\delta\chi$	$\delta\rho$	$\delta\Delta$
carga y densidad	+	+	**0**
carga y anisotropía	+	**0**	-
carga, anisotropía y densidad con $\delta\Delta/\delta\rho > 0$ y $\delta\chi/\delta\rho > 0$	+	+	+
carga, anisotropía y densidad con $\delta\Delta/\delta\rho > 0$ y $\delta\chi/\delta\rho < 0$	-	+	+
carga, anisotropía y densidad con $\delta\Delta/\delta\rho < 0$ y $\delta\chi/\delta\rho > 0$	+	+	-

Modelos y Perturbaciones de carga, anisotropía y densidad

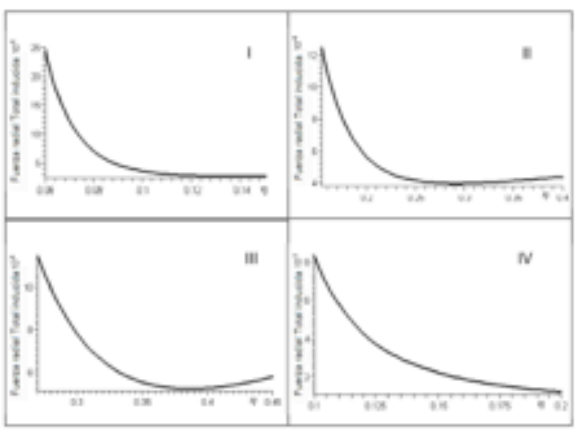
Perfil densidad	M/a	$M(M_\odot)$	z_a	$\rho_a \times 10^{14}\,(gr/cm^3)$	$\rho_c \times 10^{15}\,(gr/cm^3)$
Tolman VI	0.21	1.42	0.31	2.30	NA
NL Stewart 1	0.32	2.15	0.65	6.80	1.91
NL Stewart 2	0.39	2.68	1.19	8.49	2.14
Gokhroo & Mehra	0.26	1.76	0.44	0.00	2.09



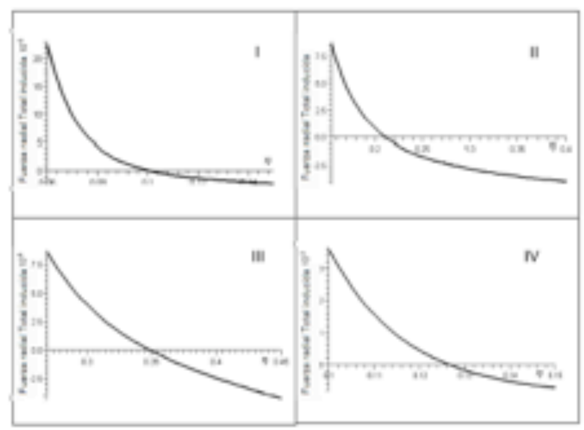
$\delta\tilde{\chi} > 0$ y $\delta\tilde{\rho} > 0$



$\delta\tilde{\chi} > 0$ y $\delta\tilde{\rho} < 0$



$\delta\tilde{\chi} < 0$ y $\delta\tilde{\rho} > 0$



$\delta\tilde{\chi} < 0$ y $\delta\tilde{\rho} < 0$

Coordenadas comóviles

$$ds_-^2 = -e^{2\sigma(r,t)}\mathrm{d}t^2 + e^{2\omega(r,t)}\mathrm{d}r^2 + R^2(r,t)(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2),$$

$$T^{\nu\mu} = (\rho + P_t)\, V^\mu V^\nu - P_t g^{\mu\nu} + (P_r - P_t)\, \chi^\mu \chi^\nu + \frac{1}{4}\pi \left[F^{\mu\alpha} F^\nu_\alpha - \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right]$$

$$\text{con } V^\alpha V_\alpha = -1, \quad \chi^\alpha \chi_\alpha = 1, \quad \chi^\alpha V_\alpha = 0$$

$$V^\alpha = e^{-\sigma(r,t)}\delta_0^\alpha, \text{ y } \chi^\alpha = e^{-\omega(r,t)}\delta_1^\alpha$$

$$D_T = e^{-\sigma(r,t)}\frac{\partial}{\partial t},\ D_R \equiv \frac{\partial}{\partial R} = \frac{1}{R'}\frac{\partial}{\partial r},\ \text{y } U = D_T R$$

$$m(\rho, R, Q) = \int_0^R 4\pi \bar{R}^2 \rho \, \mathrm{d}\bar{R} + \frac{Q^2}{2R} + \frac{1}{2}\int_0^R \frac{Q^2}{\bar{R}^2}\, \mathrm{d}\bar{R}$$

Fracturas y perturbaciones geométricas

$$D_T U = -\frac{\left(1+U^2-\frac{2m}{R}+\frac{Q^2}{R^2}\right)}{(\rho+P_r)}\left\{D_R P_r - \frac{Q}{4\pi R^4}D_R Q + 2\frac{P_r-P_t}{R}\right\} - \left(\frac{m}{R^2}-\frac{Q^2}{R^3}+4\pi P_r R\right)$$

$$\tilde{\mathcal{R}} = D_R\, P_r - \frac{Q D_R\, Q}{4\pi R^4} + \frac{(\rho+P_r)\left(\frac{m}{R^2}-\frac{Q^2}{R^3}+4\pi P_r R\right)}{\left(1+U^2-\frac{2m}{R}+\frac{Q^2}{R^2}\right)} - 2\frac{\Delta}{R}$$

$$D_T U \propto \tilde{\mathcal{R}}_0\left(\rho, m.P_r, \xi, \chi, \Delta, R\right) + \\ + \underbrace{\delta\rho\frac{\partial\tilde{\mathcal{R}}}{\partial\rho} + \delta P_r\frac{\partial\tilde{\mathcal{R}}}{\partial P_r} + \delta m\frac{\partial\tilde{\mathcal{R}}}{\partial m} + \delta\xi\frac{\partial\tilde{\mathcal{R}}}{\partial\xi} + \delta\chi\frac{\partial\tilde{\mathcal{R}}}{\partial\chi} + \delta\Delta\frac{\partial\tilde{\mathcal{R}}}{\partial\Delta} + \delta R\frac{\partial\tilde{\mathcal{R}}}{\partial R}}_{\delta\tilde{\mathcal{R}}},$$

Fracturas y perturbaciones geométricas

$$m(\rho+\delta\rho, R+\delta R, \chi+\delta\chi) \approx m(\rho, R) +$$

$$+ \underbrace{\frac{4\pi}{3} r^3 \delta\rho + \left(8\pi \int_0^R d\bar{R}\, \rho\, \bar{R} - \frac{\chi}{2R} - \int_0^R d\bar{R}\, \frac{\chi}{\bar{R}^3}\right) \delta R}_{\delta m}$$

$$\chi(\xi+\delta\xi, R+\delta R) = 2\int_0^R (\xi+\delta\xi)\, \mathrm{d}\bar{R} \approx 2\chi(\xi, R) + 2R\delta\xi \Rightarrow \delta\chi = 2R\,\delta\xi$$

$$\delta\tilde{\mathcal{R}} = \delta\Delta \frac{\partial\tilde{\mathcal{R}}}{\partial\Delta} + \delta\chi\left(\frac{1}{2R}\frac{\partial\tilde{\mathcal{R}}}{\partial\xi} + \frac{\partial\tilde{\mathcal{R}}}{\partial\chi}\right) + \delta\rho\left(\frac{\partial\tilde{\mathcal{R}}}{\partial\rho} + \frac{\partial P_r}{\partial\rho}\frac{\partial\tilde{\mathcal{R}}}{\partial P_r} + \frac{4\pi R^3}{3}\frac{\partial\tilde{\mathcal{R}}}{\partial m}\right) +$$

$$+\delta R\left(\frac{\partial\tilde{\mathcal{R}}}{\partial R} + \frac{\partial\tilde{\mathcal{R}}}{\partial m}\left(\mathcal{I}_{R1} - \frac{\chi}{2R} - \mathcal{I}_{R2}\right)\right) \; .$$

con

$$\mathcal{I}_{R1} = 8\pi\int_0^R d\bar{R}\, \rho\, \bar{R} \qquad \text{y} \qquad \mathcal{I}_{R2} = \int_0^R d\bar{R}\, \frac{\chi}{\bar{R}^3}$$

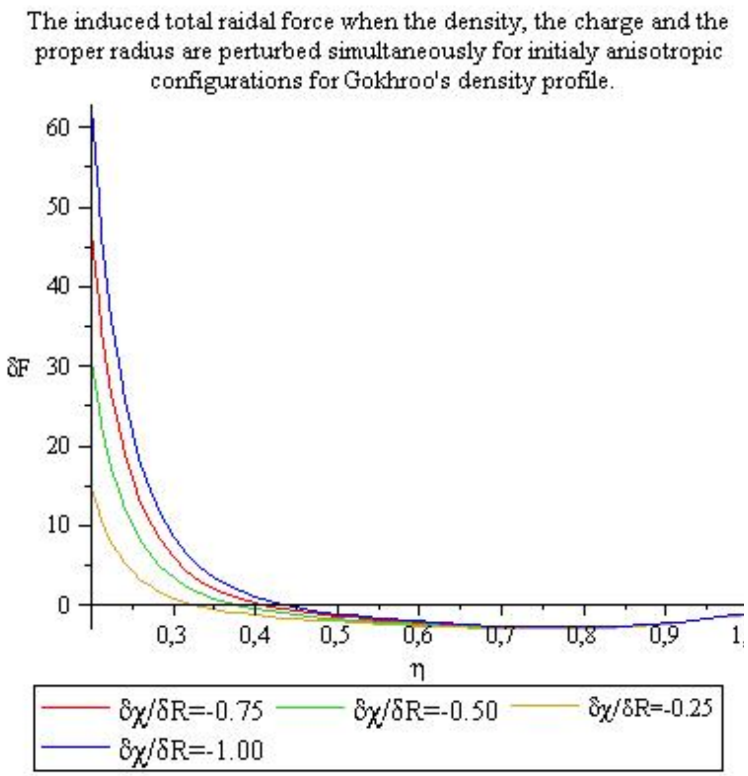
Fracturas y perturbaciones geométricas

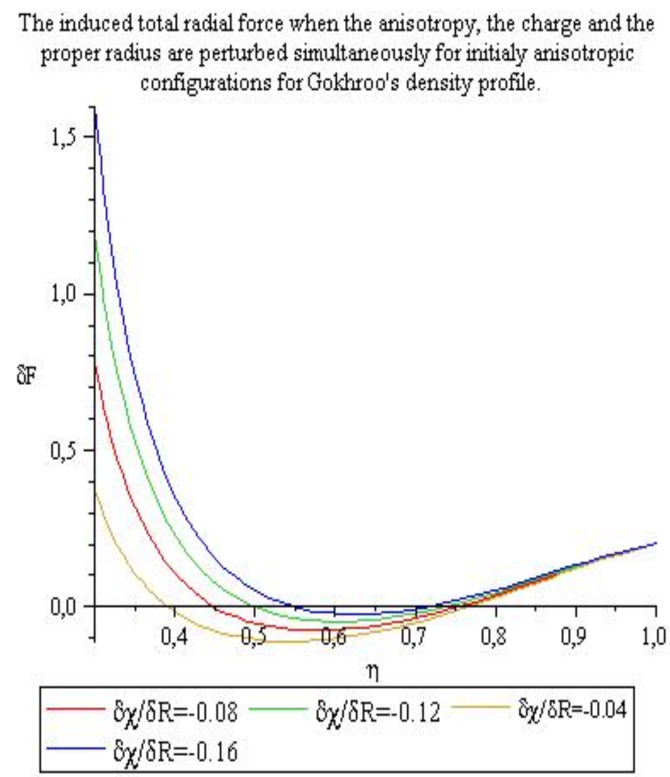
$$
\begin{aligned}
\delta\mathcal{R} = & -\frac{2\delta\Delta}{R} - \left(\frac{1}{8\pi R^5} + \frac{(\rho_0 + P_{r0})\left(4\pi R^2 P_{r0} + 1 - \frac{m_0}{R}\right)}{\left(1 - \frac{2m_0}{R}\right)^2 R^3} \right) \delta\chi \\
& + \left(\frac{4\pi}{3} \frac{(\rho_0 + P_{r0})(1 + 8\pi R^2 P_{r0})}{R^2 \left(1 - \frac{2m_0}{R}\right)^2} + \frac{\left(8\pi R P_{r0} + \frac{2m_0}{R^2}\right)}{\left(1 - \frac{2m_0}{R}\right)} \right) \delta\rho \\
& - \left(\frac{(\rho + P_{r0})\left(4\pi R^2 P_{r0} - \frac{2m_0}{R}\right)}{R^2 \left(1 - \frac{2m_0}{R}\right)} + \frac{2m(\rho_0 + P_{r0})\left(\frac{m_0}{R^2} + 4\,\pi\, P_{r0} R\right)}{\left(1 - 2\,\frac{m_0}{R}\right)^2 R^2} \right. \\
& \left. + \frac{2\Delta_0}{R^2} - \frac{\mathcal{I}_{0R}}{R^3} \left(\frac{\rho_0 + P_{r0}}{1 - \frac{2m_0}{R}} \right) \left(R + \frac{2\left(4\pi R^3 P_{r0} + m\right)}{1 - \frac{2m_0}{R}} \right) \right) \delta R
\end{aligned}
$$

Signos de perturbaciones

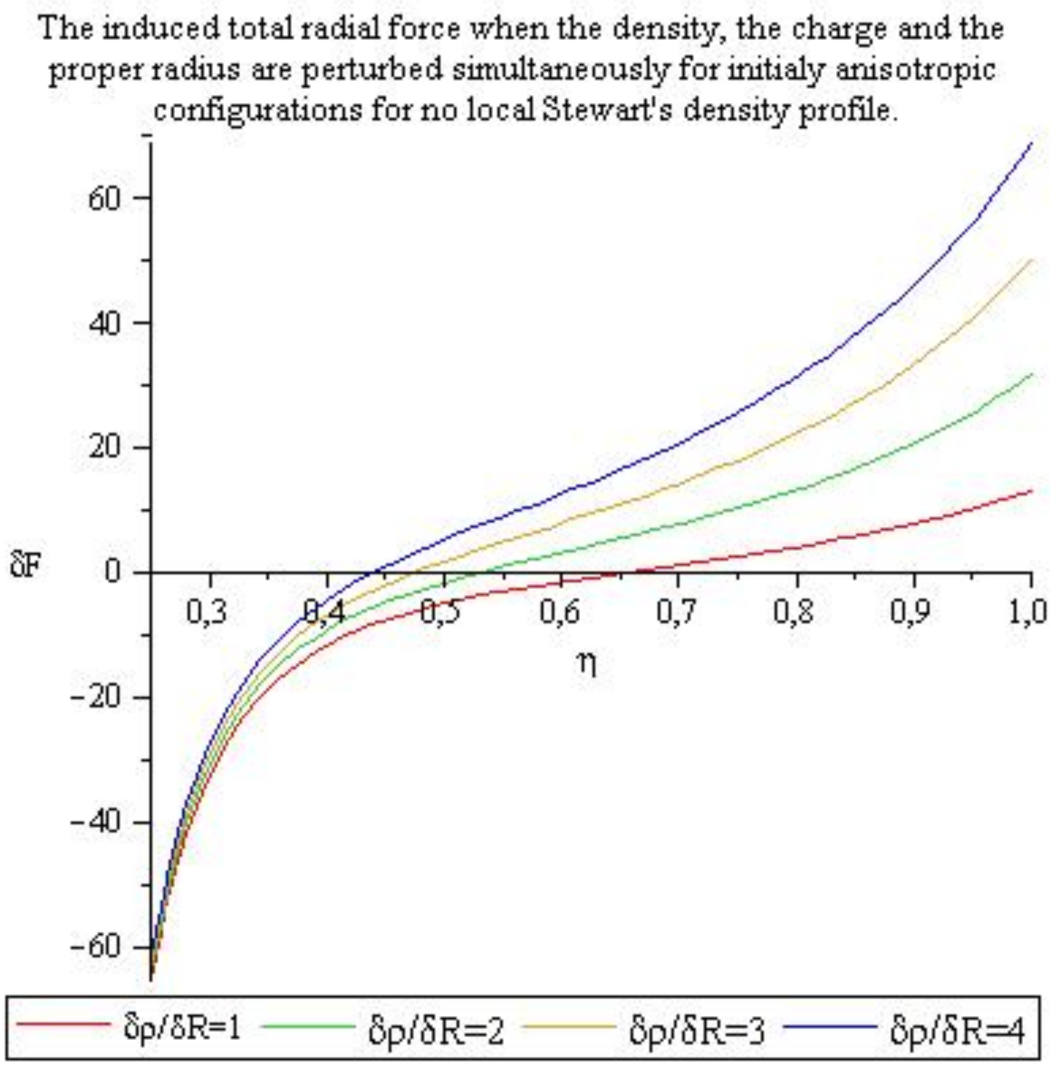
Caso	Perturbación			
	$\delta\chi$	$\delta\rho$	$\delta\Delta$	δR
Carga + Densidad	+	+	**0**	**0**
Carga + Anisotropía	+	**0**	-	**0**
Carga + Densidad + Anisotropía	+	+	+	**0**
Carga + Densidad + Anisotropía	-	+	+	**0**
Carga + Densidad + Anisotropía	+	+	-	**0**
Anisotropía + Radio Propio	**0**	**0**	+	+
Densidad + Radio Propio	**0**	+	**0**	+
Carga + Radio Propio	-	**0**	**0**	+
Carga + Anisotropía + Radio Propio	+	**0**	-	+
Carga + Anisotropía + Radio Propio	-	**0**	+	+
Carga + Densidad + Radio Propio	+	+	**0**	+
Carga + Densidad + Radio Propio	-	+	**0**	+

Gokhroo Carga y Radio Propio

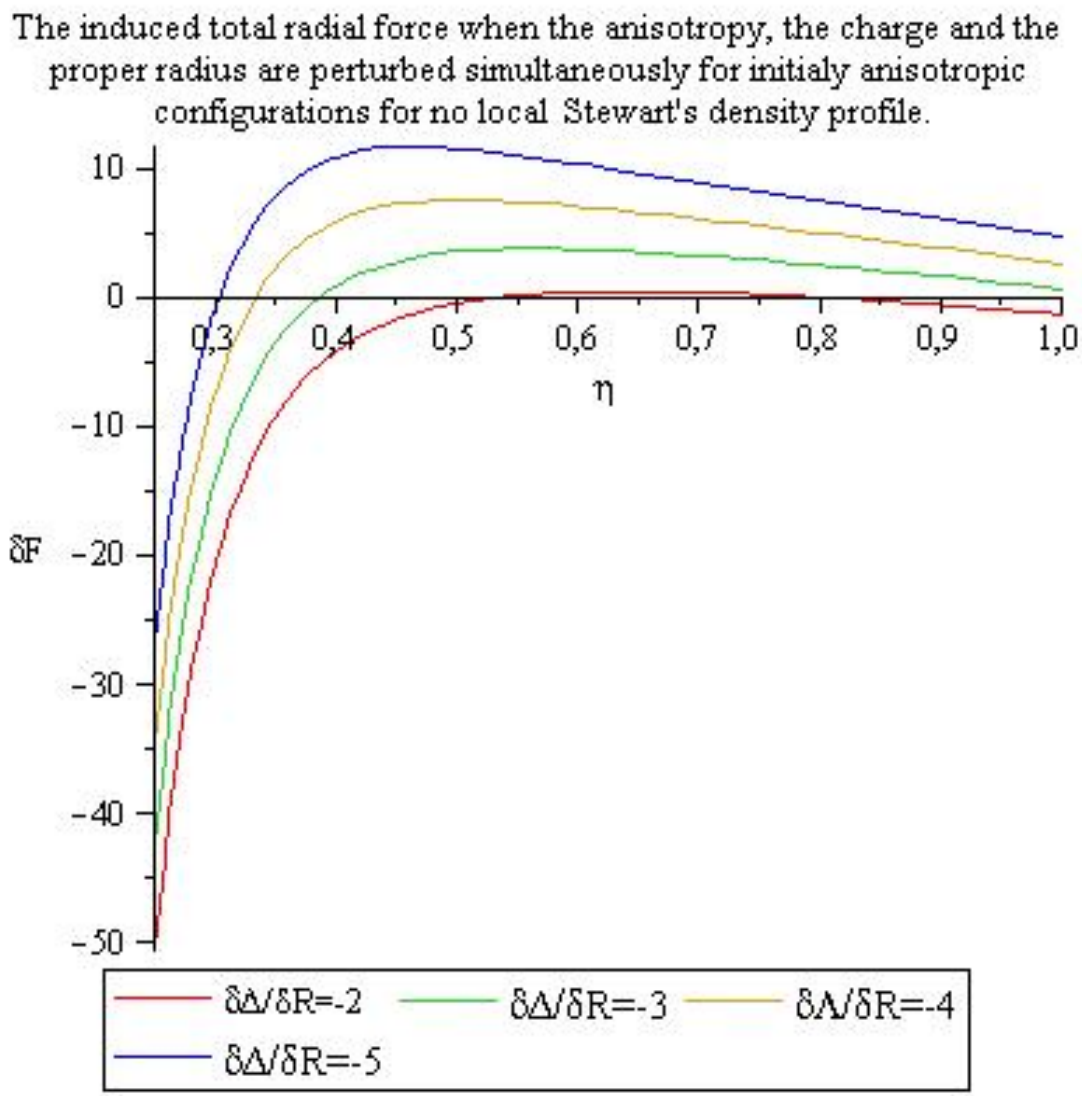




Stewart Densidad y Radio Propio

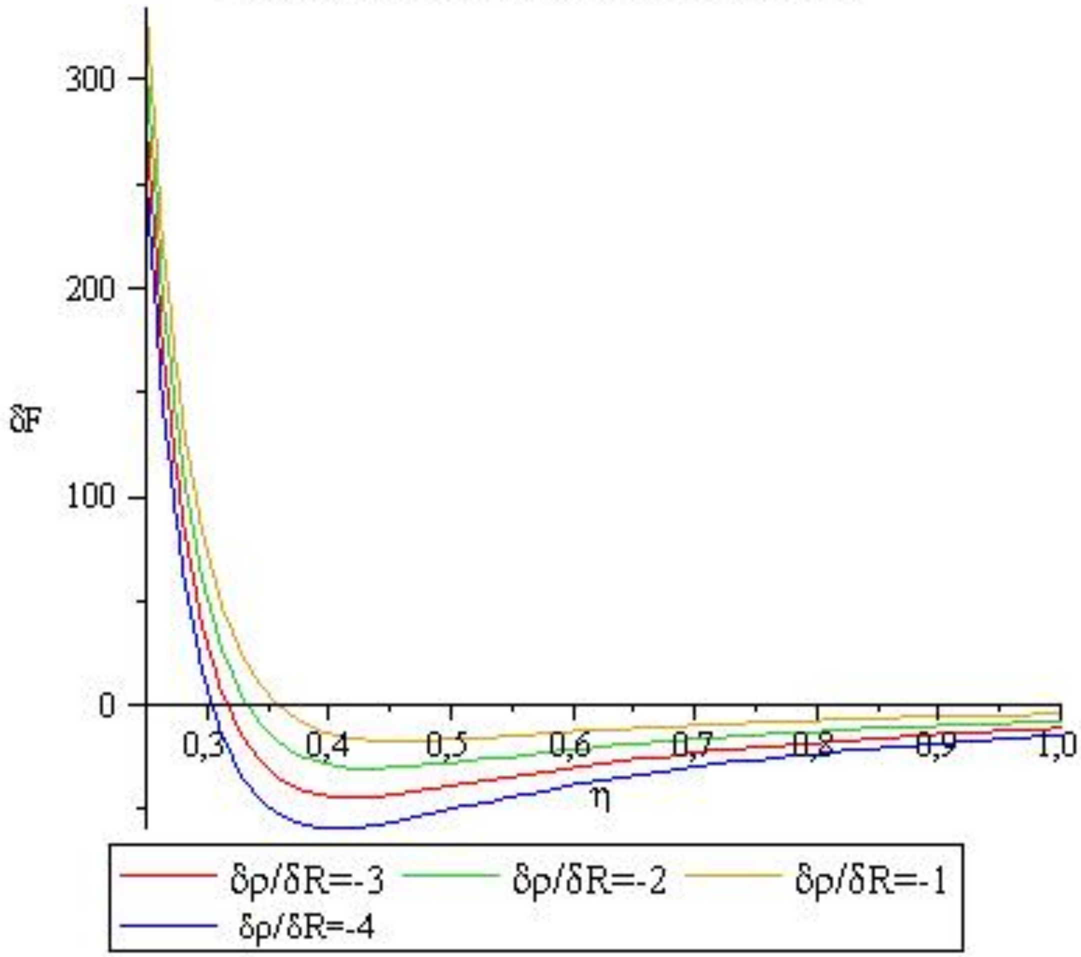


Stewart Anisotropía y Radio propio

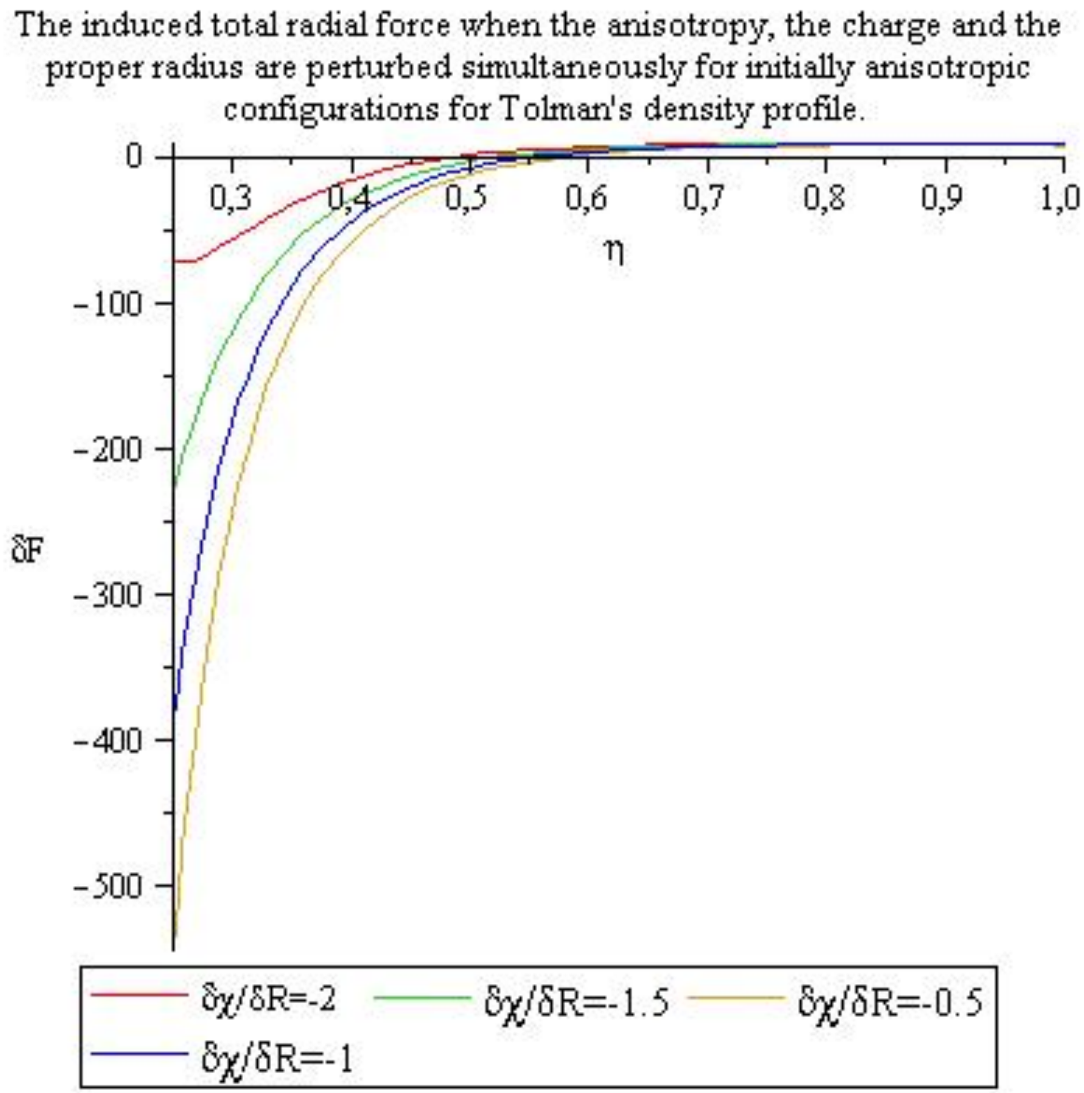


Tolman Densidad y Radio Propio

The induced total radial force when the density, the charge and the proper radius are perturbed simultaneously for initialy anisotropic configurations for Tolman's density profile.



Tolman Carga y Radio Propio



$$\mathbf{T}_{\mu\nu} = (\rho + P_\perp)\mathbf{u}_\mu \mathbf{u}_\nu - P_\perp \mathbf{g}_{\mu\nu} + (P_r - P_\perp)\mathbf{n}_\mu \mathbf{n}_\nu$$

$$G_{\mu\nu} = T\mu\nu$$

$$\mathrm{d}s^2 = e^{2\nu(r,t)}\mathrm{d}t^2 - e^{2\lambda(r,t)}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2$$

$$\mathbf{T}_r^r = -\mathbf{T}_t^t + \frac{2}{r^3}\int_0^r \bar{r}^2 \mathbf{T}_t^t \, \mathrm{d}\bar{r}$$

$$\mathrm{d}s^2 = e^{2\beta(r,t)}\left(\mathrm{d}t^2 - dr^2\right) - r^2\mathrm{d}\Omega^2$$

$$\mathbf{G}_t^t + 3\mathbf{G}_r^r + r\frac{\partial}{\partial r}\left[\mathbf{G}_t^t + \mathbf{G}_r^r\right] = 0$$

$$\Updownarrow$$

$$\mathbf{T}_r^r = -\mathbf{T}_t^t + \frac{2}{r^3}\int_0^r \bar{r}^2\mathbf{T}_t^t \,\mathrm{d}\bar{r}$$

$$\mathbf{T}_r^r = -\mathbf{T}_t^t + \frac{2}{3}\left\langle \mathbf{T}_t^t \right\rangle \qquad \left\langle \mathbf{T}_T^T \right\rangle = \frac{1}{\frac{4\pi}{3}R^3}\int_0^R 4\pi\bar{r}^2\mathbf{T}_T^T \,\mathrm{d}\bar{r}$$

$$\mathrm{d}s^2 = e^{2\beta(r,t)}\left(\mathrm{d}t^2 - dr^2\right) - r^2\mathrm{d}\Omega^2$$

$$\pounds_\xi\, \mathbf{g}_{ab} = \xi_{a;b} + \xi_{b;a} = \psi(x^\mu)\,\mathbf{g}_{ab}$$

$$\xi^\mu = \frac{1}{\mathrm{C}}\left[f(\mathrm{C}t + r) - g(\mathrm{C}t - r)\right]\delta_0^\mu + \left[f(\mathrm{C}t + r) + g(\mathrm{C}t - r)\right]\delta_1^\mu$$

$$\psi = \frac{1}{r}\left[f(\mathrm{C}t + r) + g(\mathrm{C}t - r)\right]$$

$$\mathbf{T}_{\mu\nu} = (\rho + P_\perp)\mathbf{u}_\mu \mathbf{u}_\nu - P_\perp \mathbf{g}_{\mu\nu} + (P_r - P_\perp)\mathbf{n}_\mu \mathbf{n}_\nu + \mathbf{f}_\mu \mathbf{u}_\nu + \mathbf{f}_\nu \mathbf{u}_\mu$$

$$G_{\mu\nu} = T\mu\nu$$

$$\mathrm{d}s^2 = e^{2\nu(r,t)}\mathrm{d}t^2 - e^{2\lambda(r,t)}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2$$

$$\mathbf{T}^r_r = -\mathbf{T}^t_t + \frac{2}{r^3}\int_0^r \bar{r}^2 \mathbf{T}^t_t \, \mathrm{d}\bar{r}$$

$$\mathrm{d}s^2 = e^{2\beta(r,t)}\left(\mathrm{d}t^2 - dr^2\right) - r^2\mathrm{d}\Omega^2$$

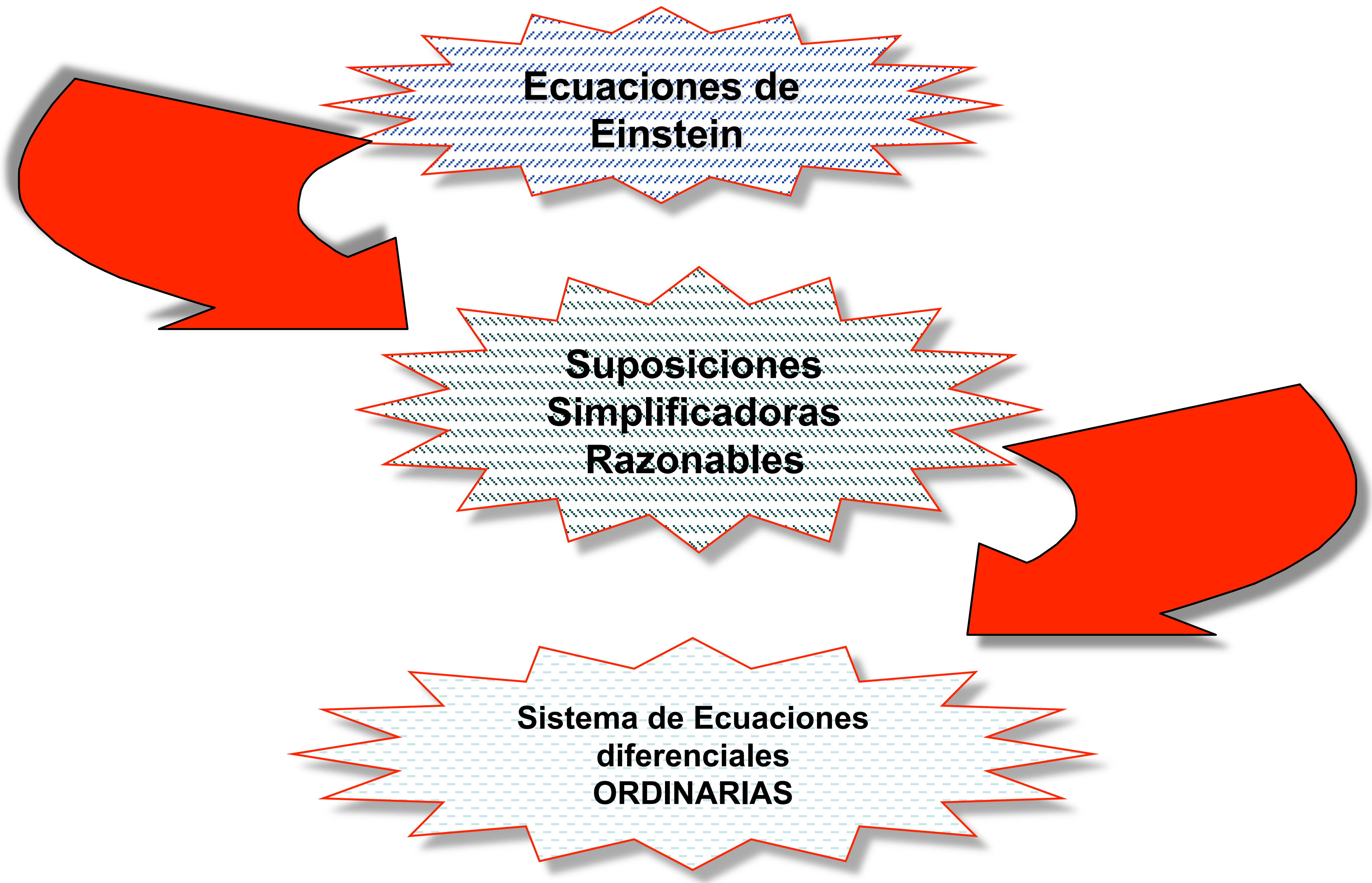
$$\mathbf{T}_{\mu\nu} = (\rho + P_\perp)\mathbf{u}_\mu \mathbf{u}_\nu - P_\perp \mathbf{g}_{\mu\nu} + (P_r - P_\perp)\mathbf{n}_\mu \mathbf{n}_\nu + \mathbf{f}_\mu \mathbf{u}_\nu + \mathbf{f}_\nu \mathbf{u}_\mu$$

$$G_{\mu\nu} = T\mu\nu$$

$$ds^2 = e^{2\gamma(u,r)} du^2 + 2e^{2\beta(u,r)}\, du\, dr - r^2 (d\vartheta^2 + \operatorname{sen}^2 \phi\, d\phi^2)$$

$$\mathbf{T}^r_r = -\mathbf{T}^t_t + \frac{2}{r^3} \int_0^r \bar{r}^2 \mathbf{T}^t_t \, \mathrm{d}\bar{r}$$

$$\mathrm{d}s^2 = e^{2\beta(r,u)} \left(\mathrm{d}u^2 + \mathrm{d}r\mathrm{d}u\right) - r^2 \mathrm{d}\Omega^2$$



Ecuaciones de Einstein
Suposiciones Simplificadoras Razonables
Sistema de Ecuaciones diferenciales ORDINARIAS

$$\mathrm{d}s^2 = e^{2\beta(r,u)} \left(\mathrm{d}u^2 + \mathrm{d}r\mathrm{d}u\right) - r^2 \mathrm{d}\Omega^2$$

Ecuaciones de Einstein

$$\left.\begin{array}{c} \rho = \rho(r) \\ P_r = P_r(r) \end{array}\right\} \Longrightarrow \left\{ \begin{array}{c} \tilde{\rho} = \rho(r)\ \mathcal{D}(u) \\ \tilde{P} = P_r(r)\ \mathcal{K}(u) \end{array}\right. .$$

Sistema de Ecuaciones diferenciales ORDINARIAS

$$h(u,r) \equiv 1 - \frac{2m(u,r)}{r} = \mathrm{C}(u)\, e^{-2\beta(u,r)},$$

$$m = 2\pi r^3 \left(\tilde{\rho} - \tilde{P}\right) \quad \Longleftrightarrow \quad \tilde{\rho} - 3\tilde{P} + r\left(\tilde{\rho}' - \tilde{P}'\right) = 0$$

$$\tilde{\rho}(r,u) = \frac{[1-\mathrm{C}(u)]\, r^2}{16\pi A^4 (2\Omega - 1)} \left[5(3-4\Omega) + 3\frac{A^2}{r^2}(8\Omega - 5)\right]$$

$$\tilde{P}(r,u) = \frac{[1-\mathrm{C}(u)]\, r^2}{16\pi A^4 (2\Omega - 1)} \left[3(3-4\Omega) + \frac{A^2}{r^2}(8\Omega - 5)\right]$$

$$\dot{A} = \mathrm{C}(\Omega - 1)\ ,$$

$$L = -\frac{1}{2}\mathrm{C}(\Omega - 1)(1-\mathrm{C})\ , \qquad \text{y}$$

$$\dot{\Omega} = \frac{2\Omega - 1}{A(1-\mathrm{C})}\left[L - \dot{A}(1-\mathrm{C})\right] - \frac{\Omega - 1}{2A}\left[1 - 11\,\mathrm{C} - 4\,\mathrm{C}\,(8\Omega - 9)\,\Omega\right]$$

Modelo Tipo Schwarzschild

$$\tilde{\rho}_{sh} = \frac{1}{8\pi}\, f\,(u) \Rightarrow \omega_a = -\frac{1}{3}\, c \quad \Leftarrow \begin{cases} \mathrm{C} = const \\ \mathrm{C} = \mathrm{C}(u) \end{cases} .$$

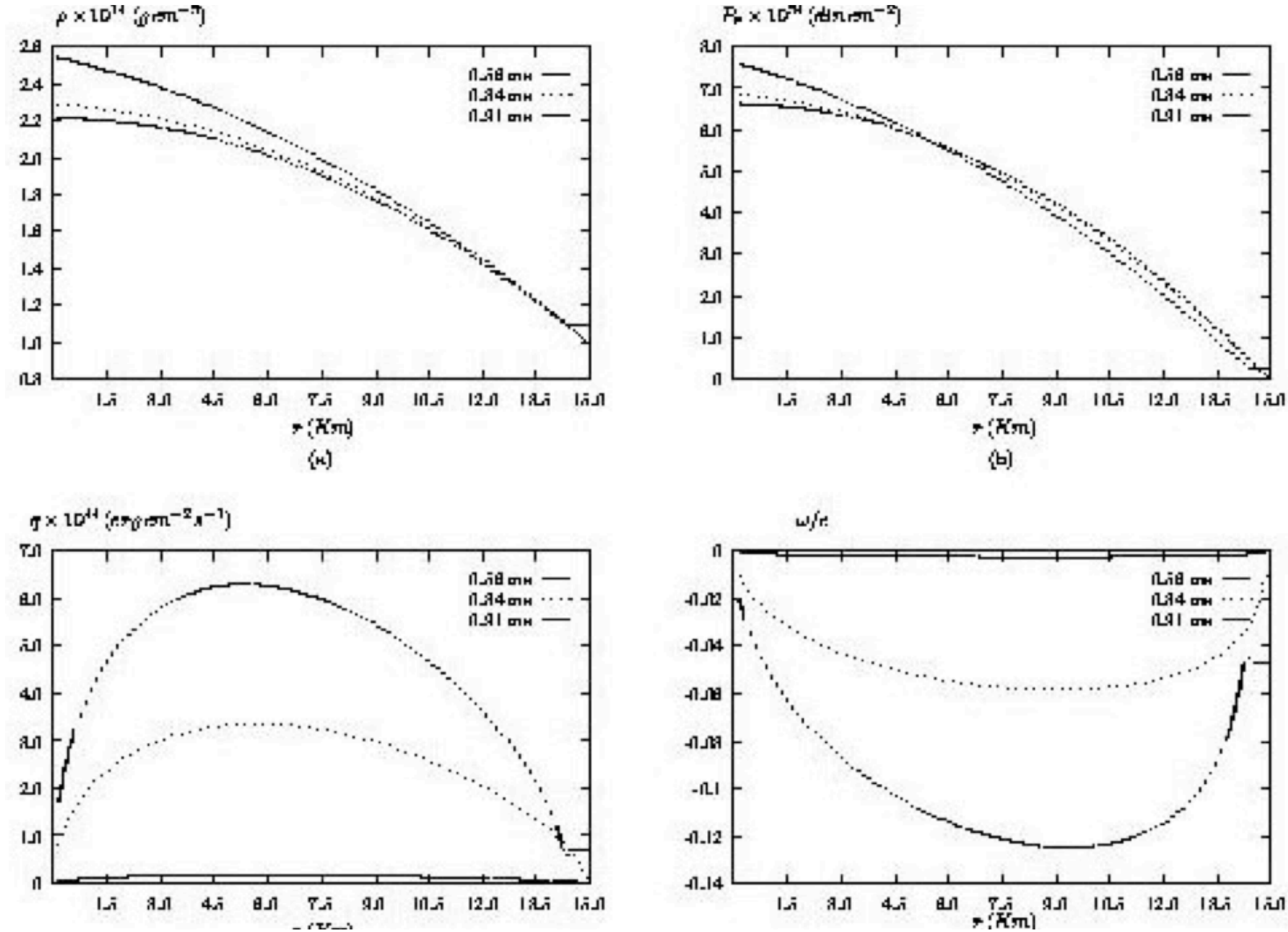
Modelo Tipo Tolman VI

$$\tilde{\rho}_{T_{VI}} = \frac{1}{8\pi r^2}\, g\,(u) \Rightarrow \omega_a = -c \quad \Leftarrow \begin{cases} \mathrm{C} = const \\ \mathrm{C} = \mathrm{C}(u) \end{cases} ..$$

Modelo Gokhroo-Mehra

$$\rho\,(r) = \rho_c \left[1 - k\frac{r^2}{a^2}\right] , \qquad \text{donde} \quad 0 \leq k \leq 1\,, \qquad \text{y}$$

$$P\,(r) = P_c \left(1 - \frac{2m\,(r)}{r}\right)\left(1 - \frac{r^2}{a^2}\right)^n , \qquad \text{con} \qquad n \geq 1\,;$$


r (Km)
(a)
(b)

$$ds^2 = e^{2\beta(r,t)}\left(dt^2 - dr^2\right) - r^2 d\Omega^2$$

Ecuaciones de Einstein

$$\left.\begin{array}{r} \rho = \rho(r) \\ P_r = P_r(r) \end{array}\right\} \Longrightarrow \left\{\begin{array}{c} \tilde{\rho} = \rho(r)\ \mathcal{D}(t) \\ \tilde{P} = P_r(r)\ \mathcal{K}(t) \end{array}\right. .$$

Sistema de Ecuaciones diferenciales ORDINARIAS

$$\rho - 3P + r\left(\rho' - P'\right) = \frac{2\left(\rho + P\right)\omega^2 + 4q\omega}{1 - \omega^2}$$

$$\Updownarrow$$

$$\mathbf{G}^t_t + 3\mathbf{G}^r_r + r\frac{\partial}{\partial r}\left[\mathbf{G}^t_t + \mathbf{G}^r_r\right] = 0$$

$$\Updownarrow$$

$$\mathbf{T}^r_r = -\mathbf{T}^t_t + \frac{2}{r^3}\int_0^r \bar{r}^2 \mathbf{T}^t_t \, \mathrm{d}\bar{r}$$

$$\Updownarrow$$

$$P = \rho - \frac{2}{r^3}\int_0^r \bar{r}^2 \left[\frac{\rho + 2q\omega + P\,\omega^2}{1 - \omega^2}\right] \mathrm{d}\bar{r}$$

$$\mathrm{d}s^2 = e^{2\lambda(r,t)}\left(\mathrm{d}t^2 - dr^2\right) - r^2\mathrm{d}\Omega^2$$

$$P(r) = \rho(r) - \frac{2}{r^3}\int_0^r \bar{r}^2\rho(\bar{r})\ \mathrm{d}\bar{r}\,.$$

$$\begin{aligned}
\rho &= \frac{e^{-2\lambda}\left[\left(2r\lambda' - e^{2\lambda} + 1\right)\omega^2 - 1 + 2r\lambda' - 4r\omega\dot{\lambda}e^{-\kappa}\right] + 1}{8\pi r^2\left(1-\omega^2\right)}\,, \\
P &= \rho - \frac{1 - e^{-2\lambda}}{4\pi r^2}\,, \\
q &= -\frac{\omega}{1+\omega^2}\left(\rho + P\right) - \left(\frac{1-\omega^2}{1+\omega^2}\right)\frac{\dot{\lambda}\,e^{-(2\lambda+\kappa)}}{4\pi r}\,, \\
P_\perp &= -\frac{e^{-2\lambda}}{8\pi}\left[\left(\ddot{\lambda} - \dot{\kappa}\dot{\lambda}\right)e^{-2\kappa} - \lambda''\right].
\end{aligned}$$

$$\left.\begin{array}{c}\rho = \rho(r) \\ P_r = P_r(r)\end{array}\right\} \Longrightarrow \left\{\begin{array}{c}\tilde{\rho} = \rho(r)\ \mathcal{D}(t) \\ \tilde{P} = P_r(r)\ \mathcal{K}(t)\end{array}\right. .$$

$$\rho - 3P + r\left(\rho' - P'\right) = \frac{2\left(\rho + P\right)\omega^2 + 4q\omega}{1 - \omega^2}$$

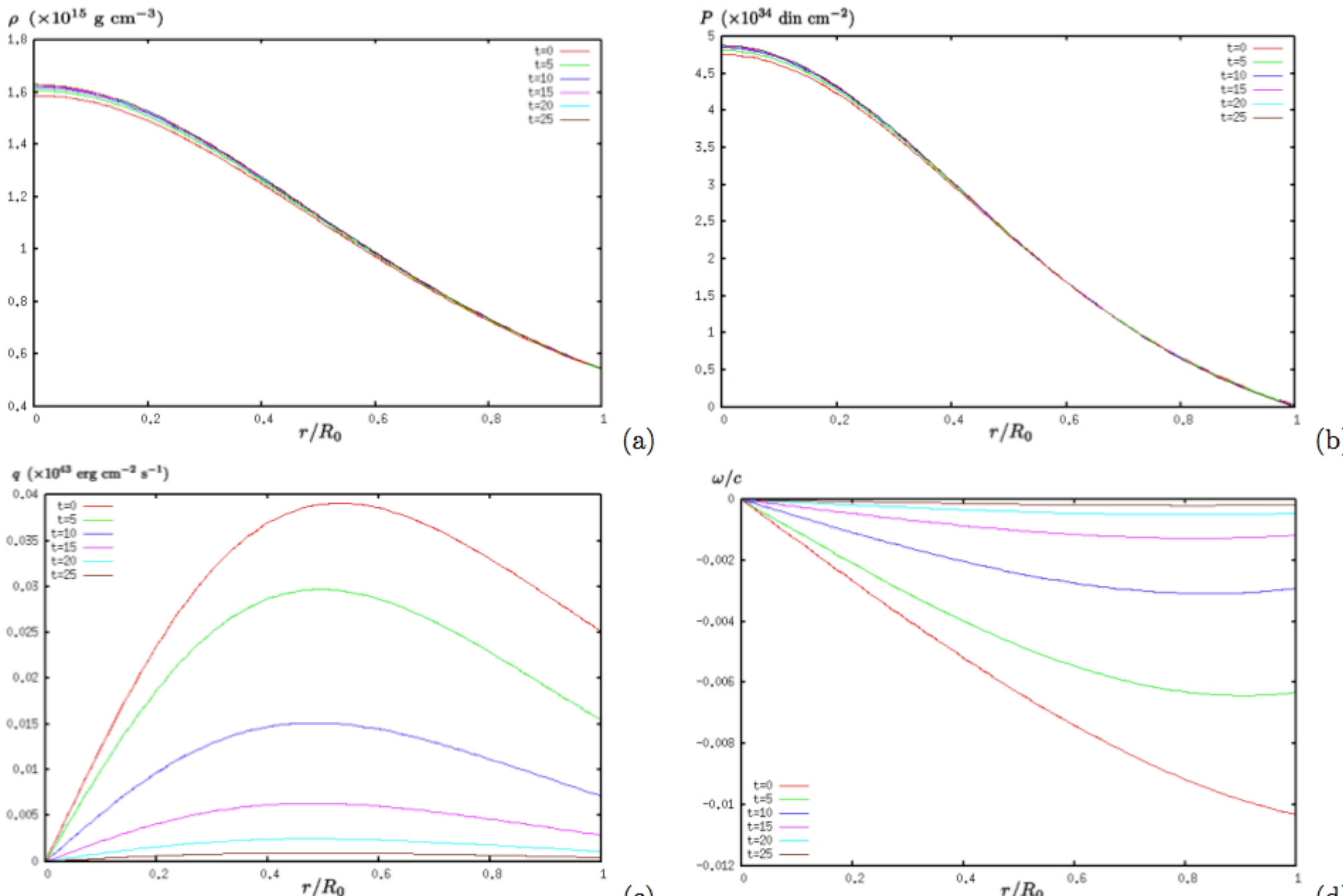
$$\rho - 3P + r\left(\rho' - P'\right) = 0 \quad = \frac{2\left(\rho + P\right)\omega^2 + 4q\omega}{1 - \omega^2}$$

$$\omega = -\frac{2q}{(\rho + P)}$$

$$
\begin{aligned}
\rho &= \frac{e^{-2\lambda}\left[\left(2r\lambda' - e^{2\lambda} + 1\right)\omega^2 - 1 + 2r\lambda' - 4r\omega\dot{\lambda}e^{-\kappa}\right] + 1}{8\pi r^2\left(1-\omega^2\right)}, \\
P &= \rho - \frac{1 - e^{-2\lambda}}{4\pi r^2}, \\
q &= -\frac{\omega}{1+\omega^2}\left(\rho + P\right) - \left(\frac{1-\omega^2}{1+\omega^2}\right)\frac{\dot{\lambda}\,e^{-(2\lambda+\kappa)}}{4\pi r}, \\
P_\perp &= -\frac{e^{-2\lambda}}{8\pi}\left[\left(\ddot{\lambda} - \dot{\kappa}\dot{\lambda}\right)e^{-2\kappa} - \lambda''\right].
\end{aligned}
$$

$$
q = \frac{2q\left(\rho+P\right)}{\left(\rho+P\right)^2 + 4q^2} - \left[\frac{\left(\rho+P\right)^2 - 4q^2}{\left(\rho+P\right)^2 + 4q^2}\right]\frac{\dot{\lambda}\,e^{-(2\lambda+\kappa)}}{4\pi r}
$$

$$
\boxed{q_1 = \frac{\dot{\lambda}\,e^{-(2\lambda+\kappa)}}{4\pi r},} \qquad q_2 = \frac{1}{2}\left(\rho + P\right), \qquad q_3 = -\frac{1}{2}\left(\rho + P\right)
$$



ρ (×10^15 g cm^−3)
r/R_0
(a)
P (×10^34 din cm^−2)
r/R_0
(b)
q (×10^43 erg cm^−2 s^−1)
r/R_0
(c)
ω/c
r/R_0
(d)
t=0
t=5
t=10
t=15
t=20
t=25

Conclusiones

- Las ecuaciones de estado no locales pueden describir objetos compactos estables anisótropos en RG
- No generan inestabilidades-estabilidad adicional
- El esquema de fractura parece eficiente para evaluar las inestabilidades potenciales de configuraciones materiales anisótropas
- Las perturbaciones pueden/deben ser acotadas con criterios físicos (velocidades del sonido)
- Pequeñas perturbaciones de carga pueden generar inestabiliades potenciales
- El esquema de fractura puede se extendido otras descripciones coordenadas.
- Las ecuaciones de estado nolocal permiten la integración analítica de un caso de colapso gravitacional Relativista